Astrophysical Radiative Processes, Spring 2025

Problem Set I

Deadline: 5PM of Thursday, March 27, 2025

1. **Thermal emission.** (15%) A supernova remnant (SNR) has an angular diameter $\theta = 4.3'$ and a flux at 100 MHz of $F_{100} = 1.6 \times 10^{-19}\,\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}}$. Assume that the emission is thermal.

   (a) (5%) What is the brightness temperature T_b? What energy regime of the blackbody curve does this correspond to?

   (b) (5%) The emitting region is actually more compact than indicated by the observed angular diameter. What effect does this have on the value of T_b?

   (c) (5%) At what frequency will the radiation of this object be maximum, if the emission is blackbody? What can you say about the temperature of this SNR?

2. **Effects of optical depth.** (20%) A certain gas emits thermally at the rate P_ν (power per unit volume per frequency interval). A spherical cloud of this gas has radius R, temperature T, and is at a distance d from earth, where $d \gg R$ (Fig. 1).

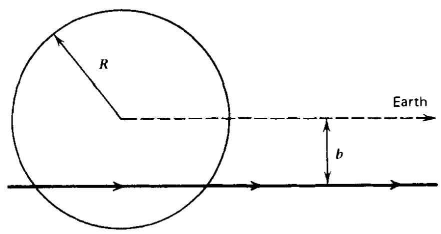

Figure 1: Detection of rays from a spherical cloud of radius R.

   (a) (3%) Assume that the cloud is optically thin. What is the intensity of the cloud observed on earth? Give your answer as a function of the distance b away from the cloud center.

   (b) (2%) What is the effective temperature of the cloud?

   (c) (2%) What is the flux F_ν measured at earth coming from the entire cloud?

(d) (3%) How do the measured brightness temperatures compare with the cloud's kinetic temperature, T?

(e) (10%) Answer parts (a)–(d) for an optically thick cloud.

3. **Effects of temperature contrast.** (10%) A spherical, opaque object emits as a blackbody at temperature T_c. Surrounding this central object is a spherical shell of thermally emitting gas at temperature T_s (Fig. 2a). The gas in the shell absorbs in a narrow spectral line; that is, its absorption coefficient becomes large at the frequency ν_0 and is negligibly small at other frequencies, such as ν_1, $\alpha_{\nu_0} \gg \alpha_{\nu_1}$ (Fig. 2b). The object is observed at frequencies ν_0 and ν_1 along two rays A and B (Fig. 2a). Assume that the Planck function does not vary appreciably between ν_0 and ν_1.

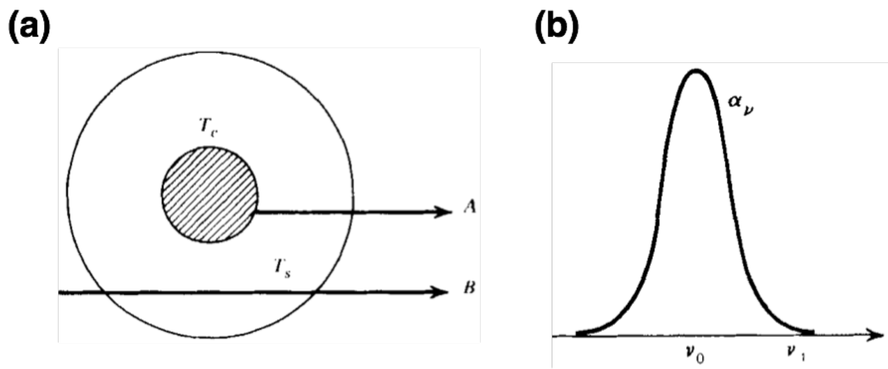

Figure 2: (a) Blackbody emitter at temperature T_c surrounded by a shell of thermally emitting gas at temperature T_s as viewed along two rays A and B. (b) Absorption coefficient of the gas in the shell.

(a) (5%) Assume $T_s < T_c$. At which frequencies will the observed intensity be larger when observing along ray A? How about along ray B?

(b) (5%) Similarly, assume $T_s > T_c$. At which frequencies will the observed intensity be larger when observing along ray A? How about along ray B?

4. **Eddington luminosity.** (15%) Radiation pressure exerted on a cloud can sometimes oppose to gravitational force and even reverts the path of motions. Consider the following situation to find a threshold set by such force.

(a) (5%) Show that an optically thin cloud can be ejected by radiation pressure from a nearby luminous object if the mass to luminosity ratio, M/L, of the object satisfies

$$\frac{M}{L} < \frac{\kappa}{4\pi G c},$$

where κ is the opacity of the cloud medium.

(b) (5%) Calculate the terminal velocity v_t attained by such a cloud under radiation and gravitational forces alone, if it starts from rest a distance R from the object. Show that

$$v_t^2 = \frac{2GM}{R}\left(\frac{\kappa L}{4\pi G M c} - 1\right).$$

(c) (5%) A minimum value for κ may be estimated for pure hydrogen as that due to Thomson scattering off free electrons, when the hydrogen is completely ionized. Let the Thomson cross section be $\sigma_T = 6.65 \times 10^{-25}$ cm^2, so the opacity is larger than $\sigma_T/m_{\rm H}$, where $m_{\rm H}$ is the mass of hydrogen atom. Show that the *Eddington luminosity*, the maximum luminosity that a central mass M can have and still not spontaneously eject hydrogen by radiation pressure, is given by

$$\begin{aligned} L_{\rm Edd} &= \frac{4\pi G M c\, m_{\rm H}}{\sigma_T} \\ &= 3.25 \times 10^4\, L_\odot \left(\frac{M}{M_\odot}\right). \end{aligned}$$