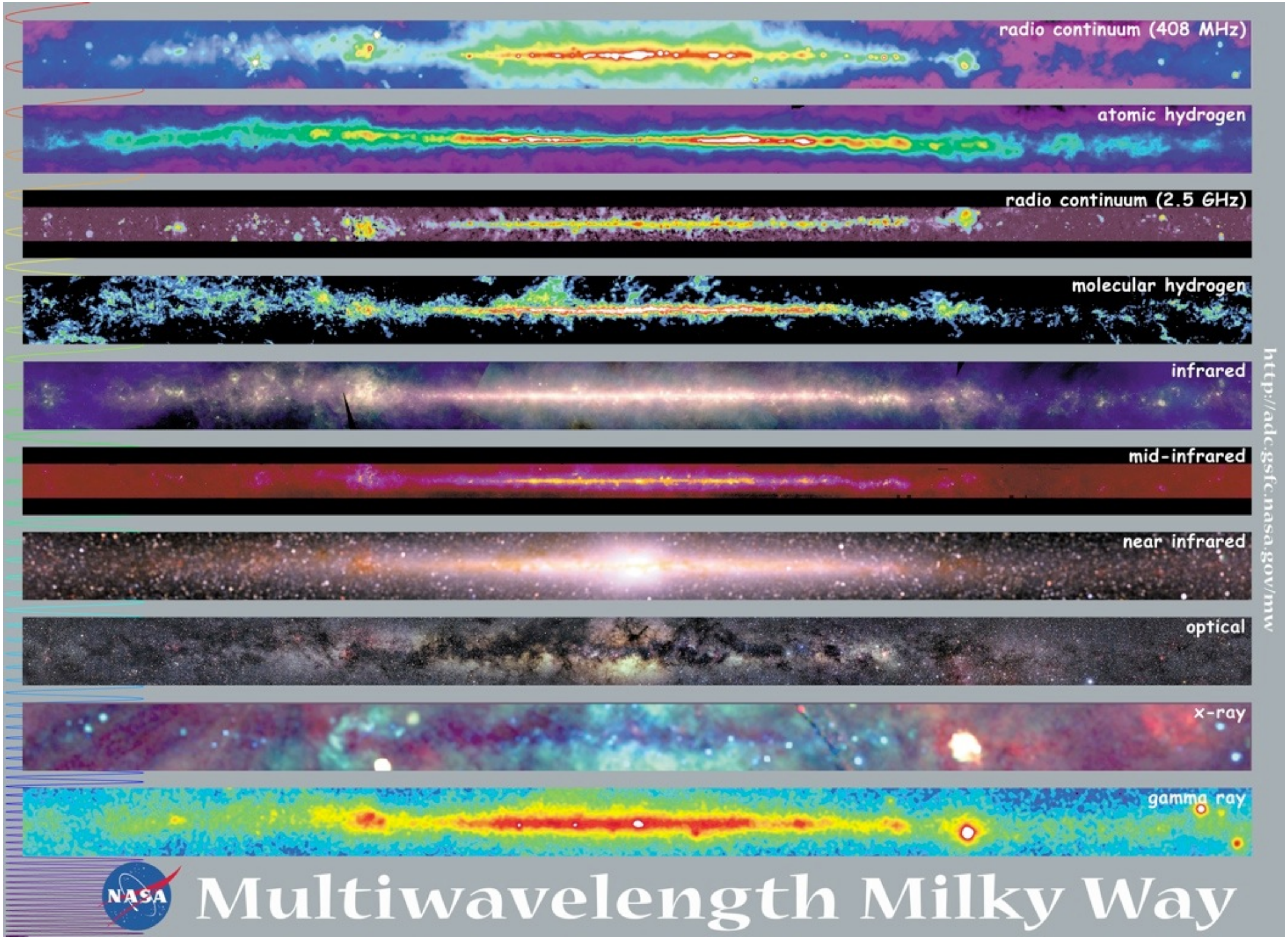
Chapter 02

Astronomical Measurements

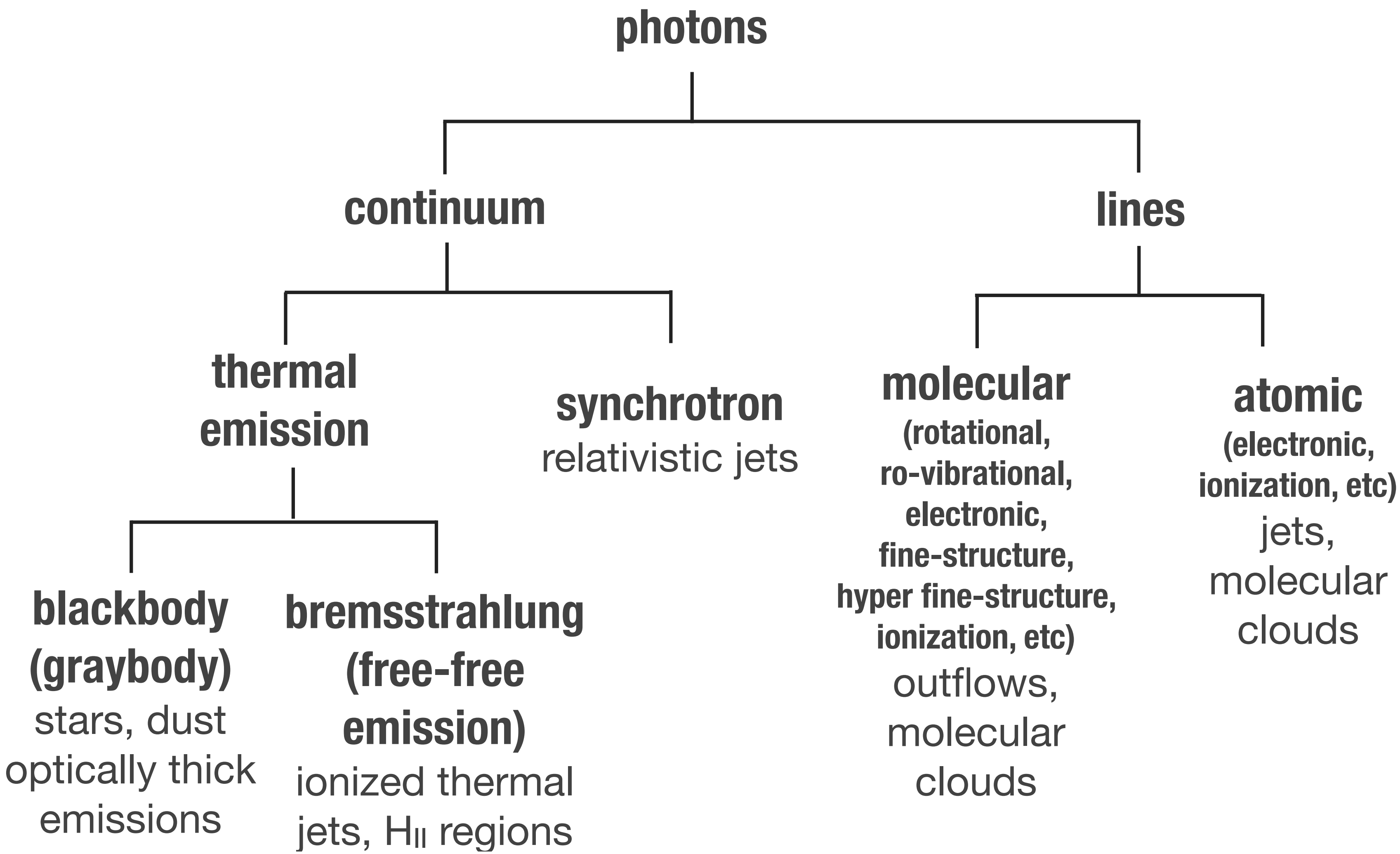
References:
CBMB: ch 1
CO: sec 1.3, 24.3
BM: sec 2.1, 2.2, 2.3, 2.5; 3.5, 3.6, 3.7

How do we “see” the Galaxy?

How do we “see” the Galaxy?

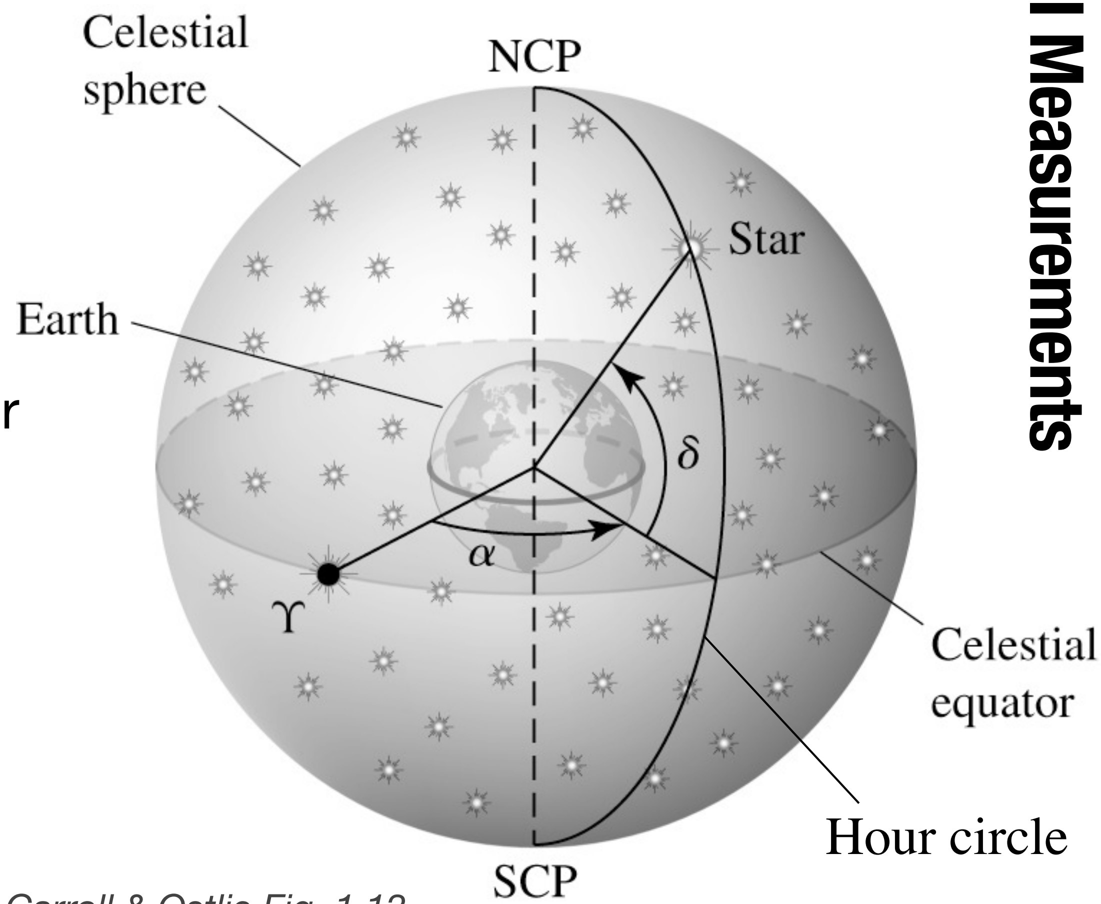


Radiative Mechanisms



Equatorial Coordinates

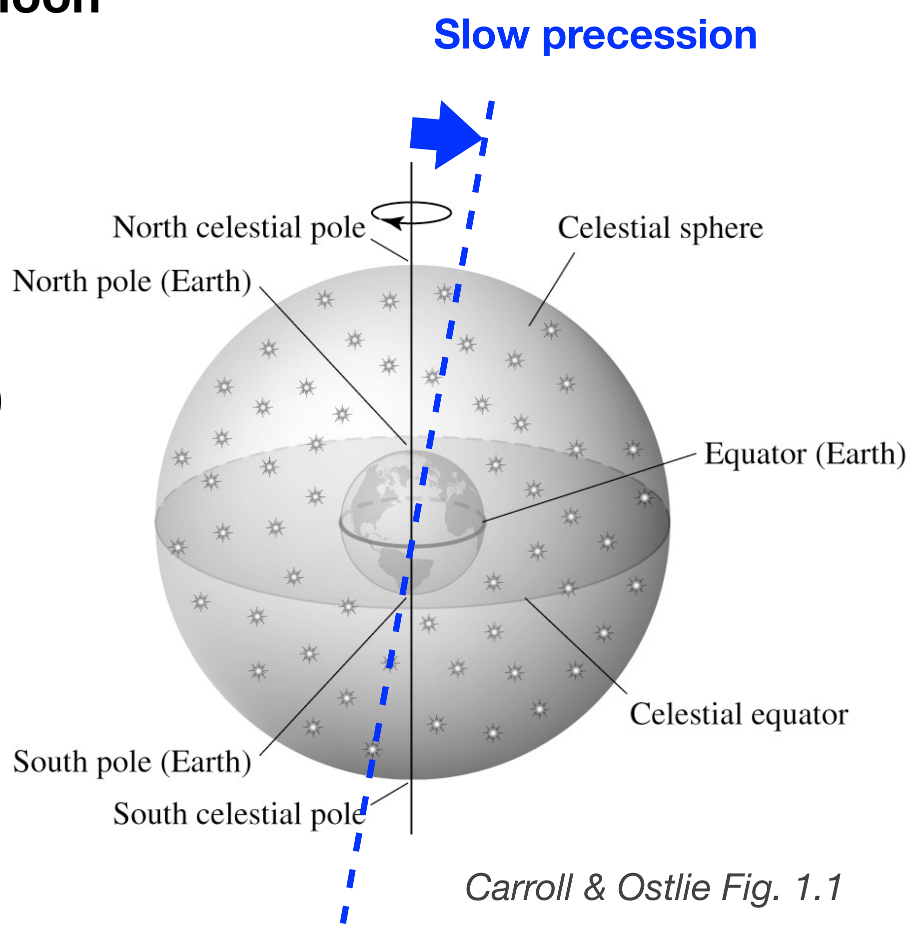
- **(α, δ) = (RA, Dec)**
- **Declination δ**
- **Right ascension α**
  - **Described by 24 hrs**
  - **1 hour = 15 deg**
- **Hour circle**
- **Local sidereal time:** elapsed time since the vernal equinox last traversed the meridian (hour angle of the vernal equinox)
- **Hour angle H**



Carroll & Ostlie Fig. 1.13

Precession & Epoch

- Precession
  - First observed by Hipparchus
  - Slow wobble of Earth's rotation axis due to its non-spherical shape and its interactions with the Sun and the Moon
- Epoch
  - Commonly used: B1950, J2000
  - Altair, the brightest star in Aquila
    - (J2000)(19h50m47.0s,+8°52′6.0″)
    - (B1950)(19h48m22.4s,+8°44′24.8″)
- ICRS
  - Latest reference frame
  - Free from the hassle of epochs



Carroll & Ostlie Fig. 1.1

Celestial Reference Systems

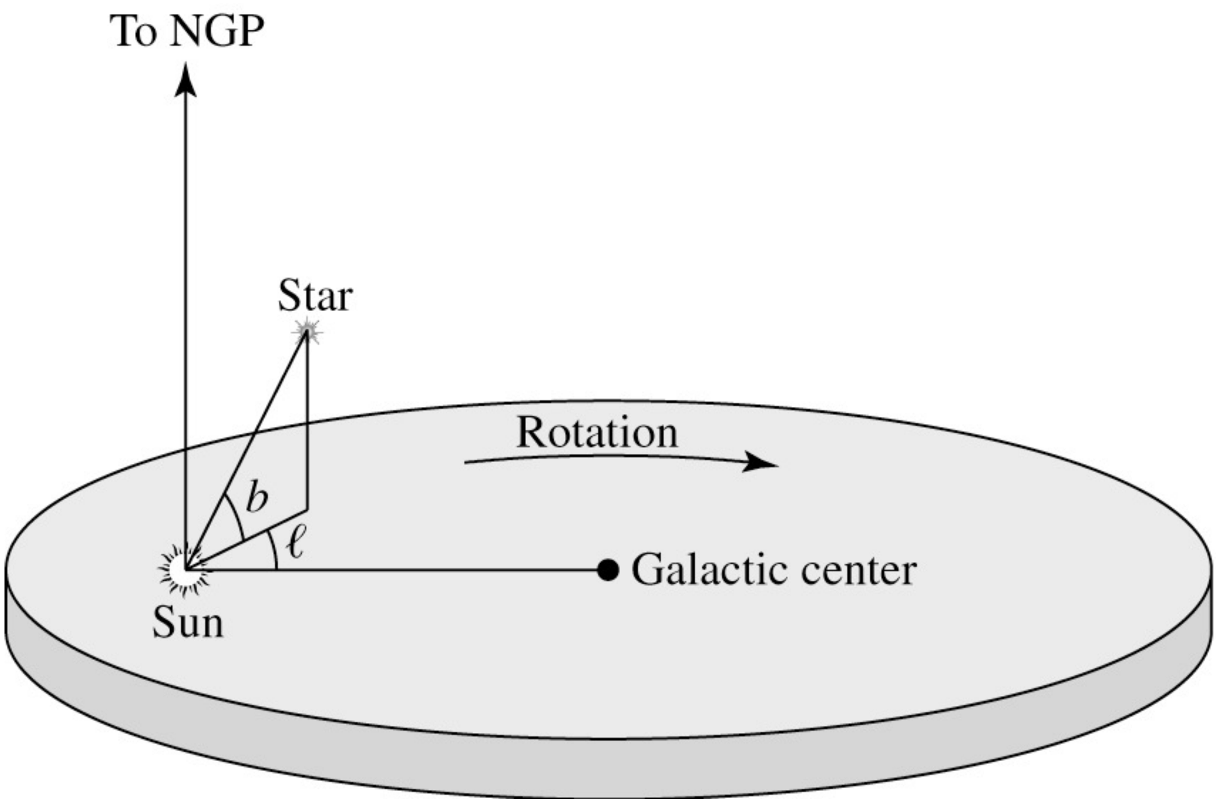
- FK5
  - astrometry based on optical catalogues
  - More CRFs: Hipparcos Celestial Reference Frame (HCRF), Third *Gaia* Celestial Reference Frame (*Gaia*-CRF3)

- ICRS (International Celestial Reference System)
  - free from the hassle of epochs
  - replacing FK5 from January 1, 1998
  - astrometry based on VLBI (radio observations)
  - accessible by means of coordinates of reference extragalactic radio sources, the International Celestial Reference Frame (ICRF)

- Useful links
  - https://www.iers.org/IERS/EN/Science/ICRS/ICRS.html
  - https://en.wikipedia.org/wiki/International_Celestial_Reference_System_and_its_realizations

Galactic Coordinates

- (l, b) = (Galactic longitude, Galactic latitude)



Carroll & Ostlie Fig. 24.17

Radial Velocities

Frequency observed by an observer moving at a velocity v_r

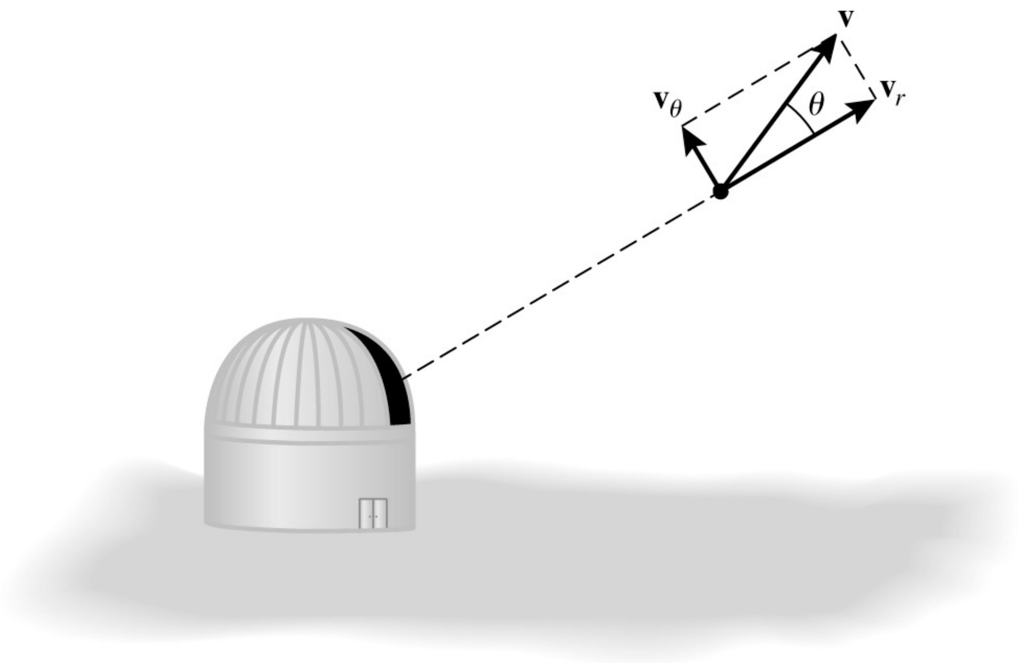
$$\nu = (1-\beta)\gamma\nu_0$$
$$\beta \equiv v_r/c$$
$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$$

Doppler effect $\gamma \to 1$

$$v_r = \frac{\Delta\lambda}{\lambda_0}c$$

Signs:

Blueshifted: $v_r < 0,\ \nu > \nu_0$
Redshifted: $v_r > 0,\ \nu < \nu_0$



Carroll & Ostlie Fig. 1.15

Magnitudes

Astronomers measure the brightness of a star with **magnitude**, which is based on human eye response to the light and therefore on a logarithmic scale. In short, 5 magnitude increment corresponds to 100 times dimmer in brightness.

Apparent magnitudes

$$m_1 - m_2 = -k \log\left(\frac{f_1}{f_2}\right)$$

$$\frac{f_1}{f_2} = 10^{-0.4(m_1 - m_2)}$$

Absolute magnitudes

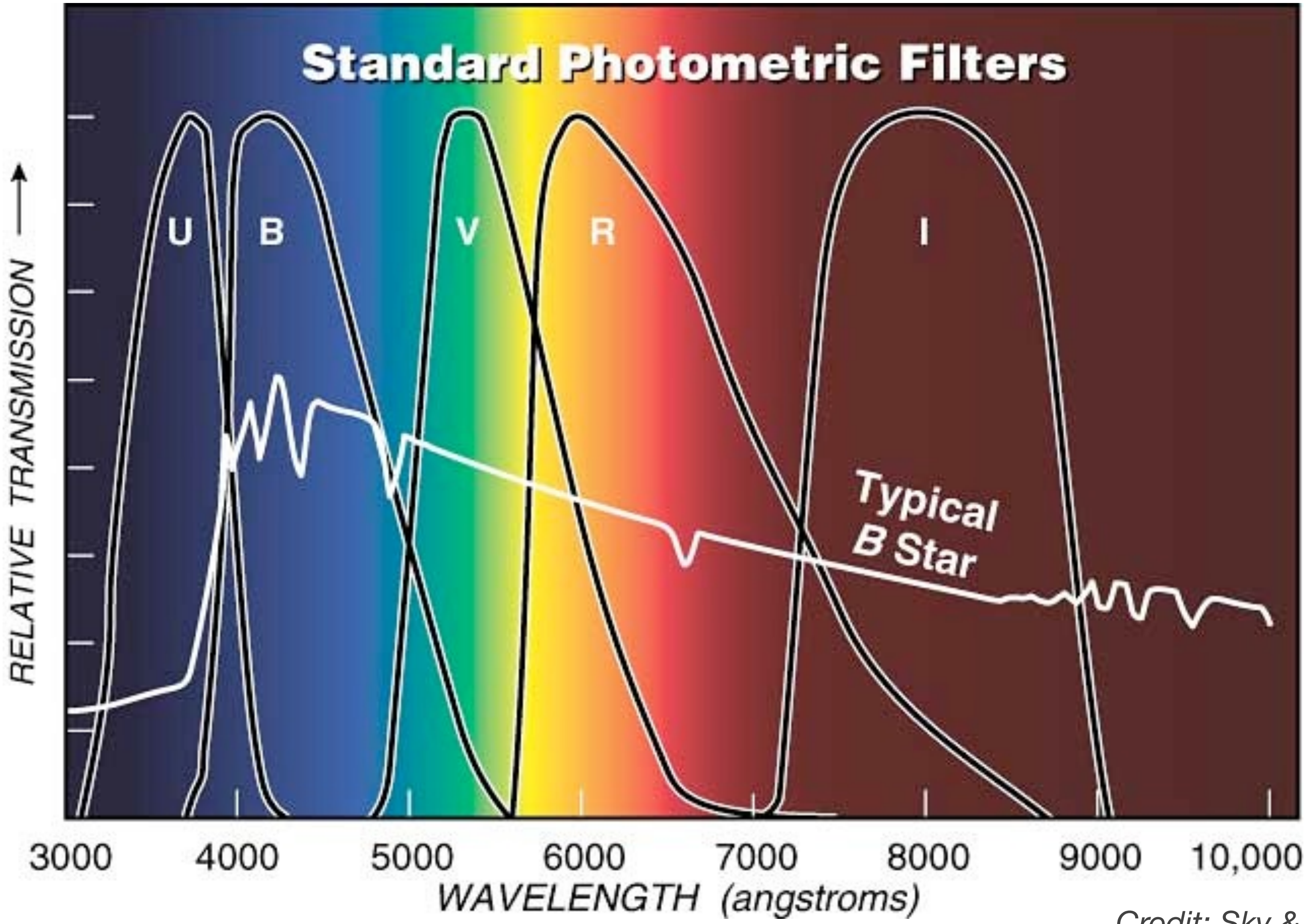
$$m - M = 5 \log d - 5, \quad \text{distance modulus}$$

$$f = \left(\frac{D}{d}\right)^2 F, \quad \text{where } D = 10 \text{ pc}$$

$$m - M = -2.5 \log\left(\frac{f}{F}\right) = 5 \log\left(\frac{d}{D}\right)$$

Filters and Colors

- *U B V R I J H K L M*

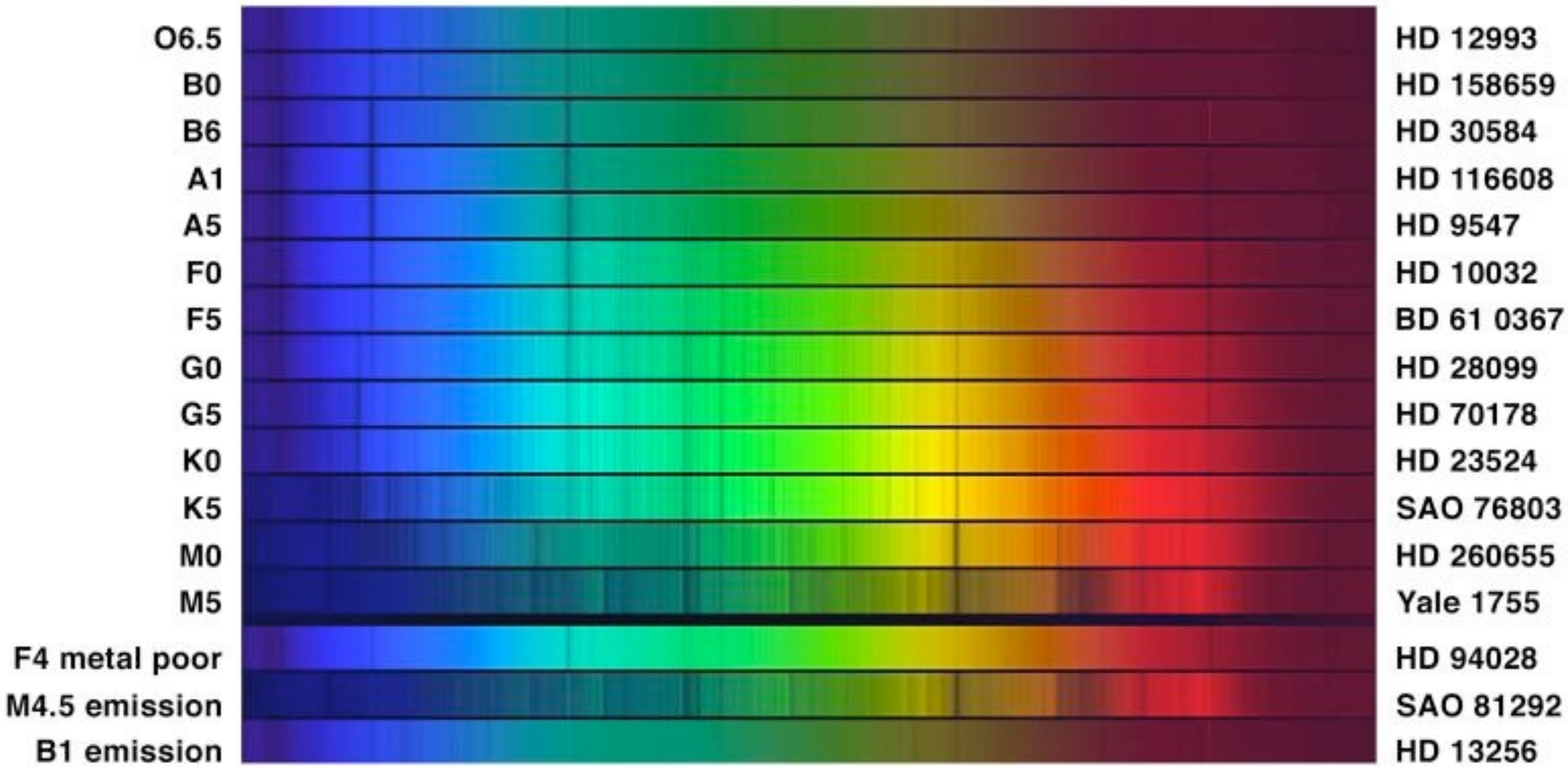


Credit: Sky & Telescope

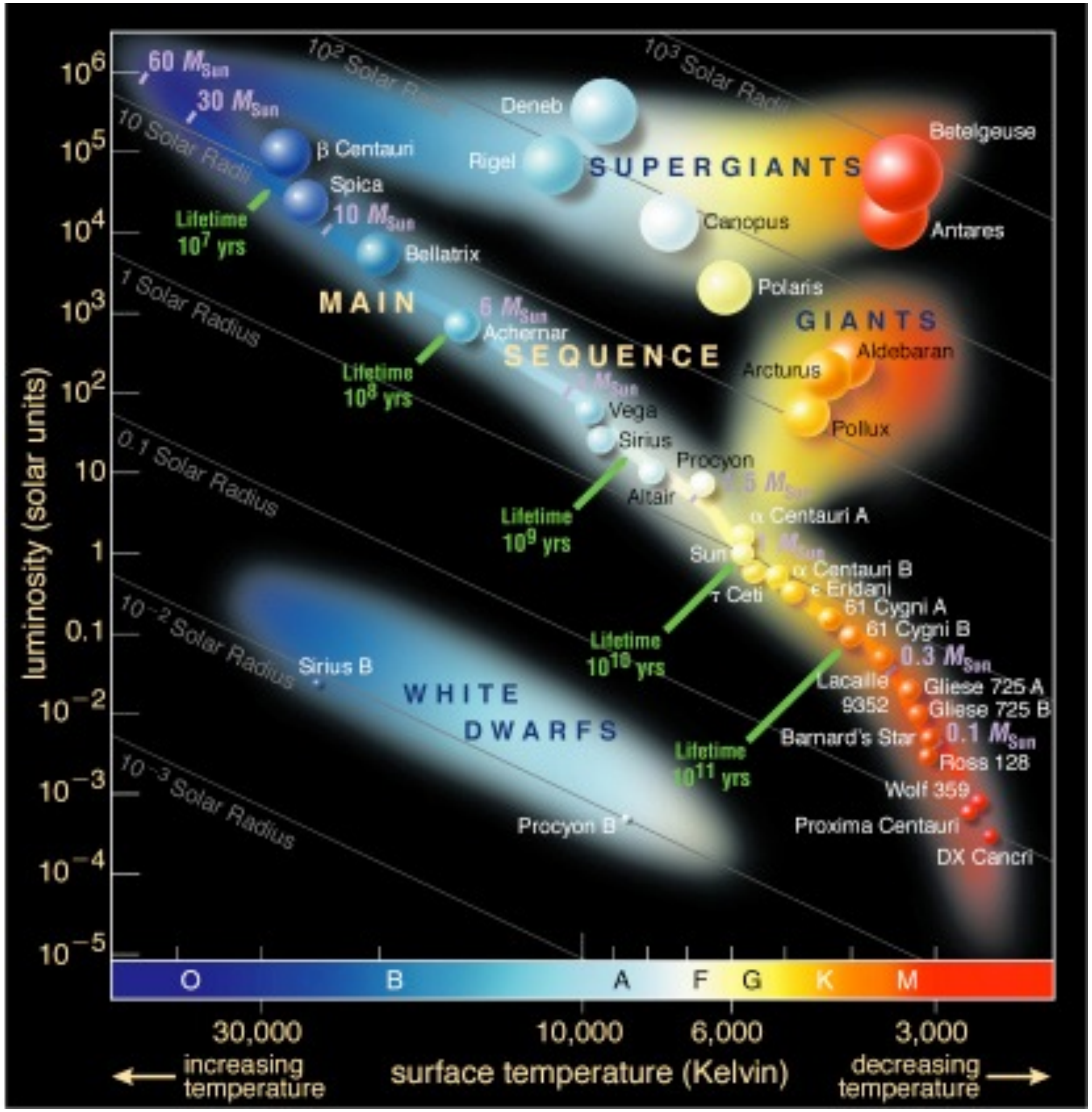
Stellar Spectral Types

- **(early) O B A F G K M L T Y (late)**
  - Stars (a.k.a. dwarfs): OBAFGKM
  - Brown dwarfs: LTY
  - Followed by subclass 0, 1, ..., 9

Credit: KPNO 0.9-m Telescope, AURA, NOAO, NSF



Luminosity Classes



- **Morgan-Keenan (MK) luminosity class**
  - I. supergiants
  - II. bright giants
  - III. normal giants
  - IV. subgiants
  - V. dwarfs = main sequence

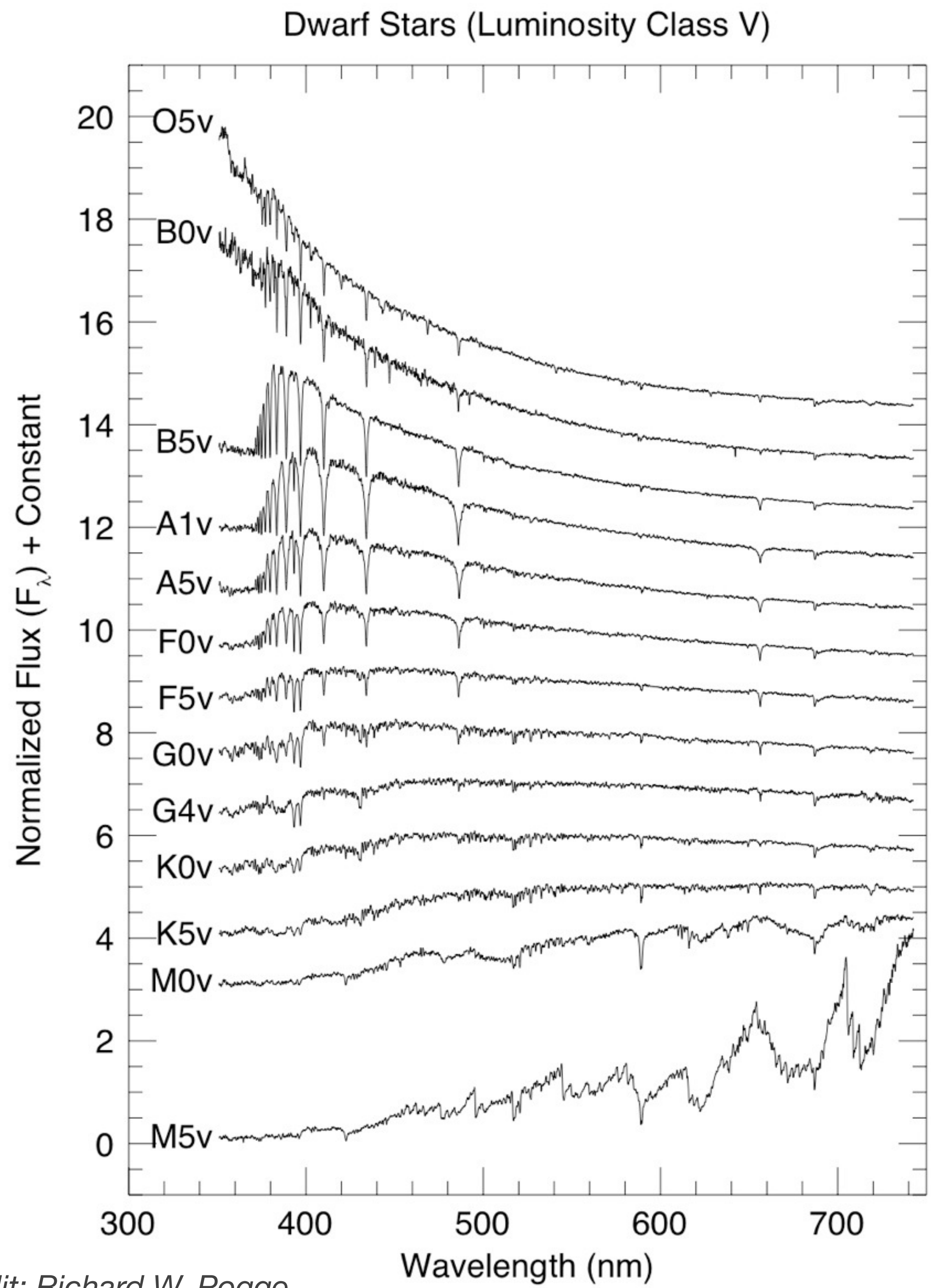
Colors

With more than one filter bands, we can take the difference in magnitudes measured in two different bands to form a **color**, or **color index** for a selected source. Let X and Y denote two different filters, a color can be obtained with

$$\begin{aligned} X - Y &\equiv m_X - m_Y \\ &= \text{const.} - 2.5 \log \frac{\int_0^\infty \mathrm{d}\lambda \, S_\lambda(X) f_\lambda}{\int_0^\infty \mathrm{d}\lambda \, S_\lambda(Y) f_\lambda} \end{aligned}$$

A **color** can serve as a measure of stellar temperature when the amount of extinction is known.

Stellar Spectral Types



Credit: Richard W. Pogge

Interpretation of Stellar Spectra

Effective temperature T_{eff}:

$$T_{\text{eff}} \equiv \left(\frac{L}{4\pi\, R^2 \sigma_{\text{SB}}} \right)^{1/4} .$$

Surface gravity: pressure-broadening lines

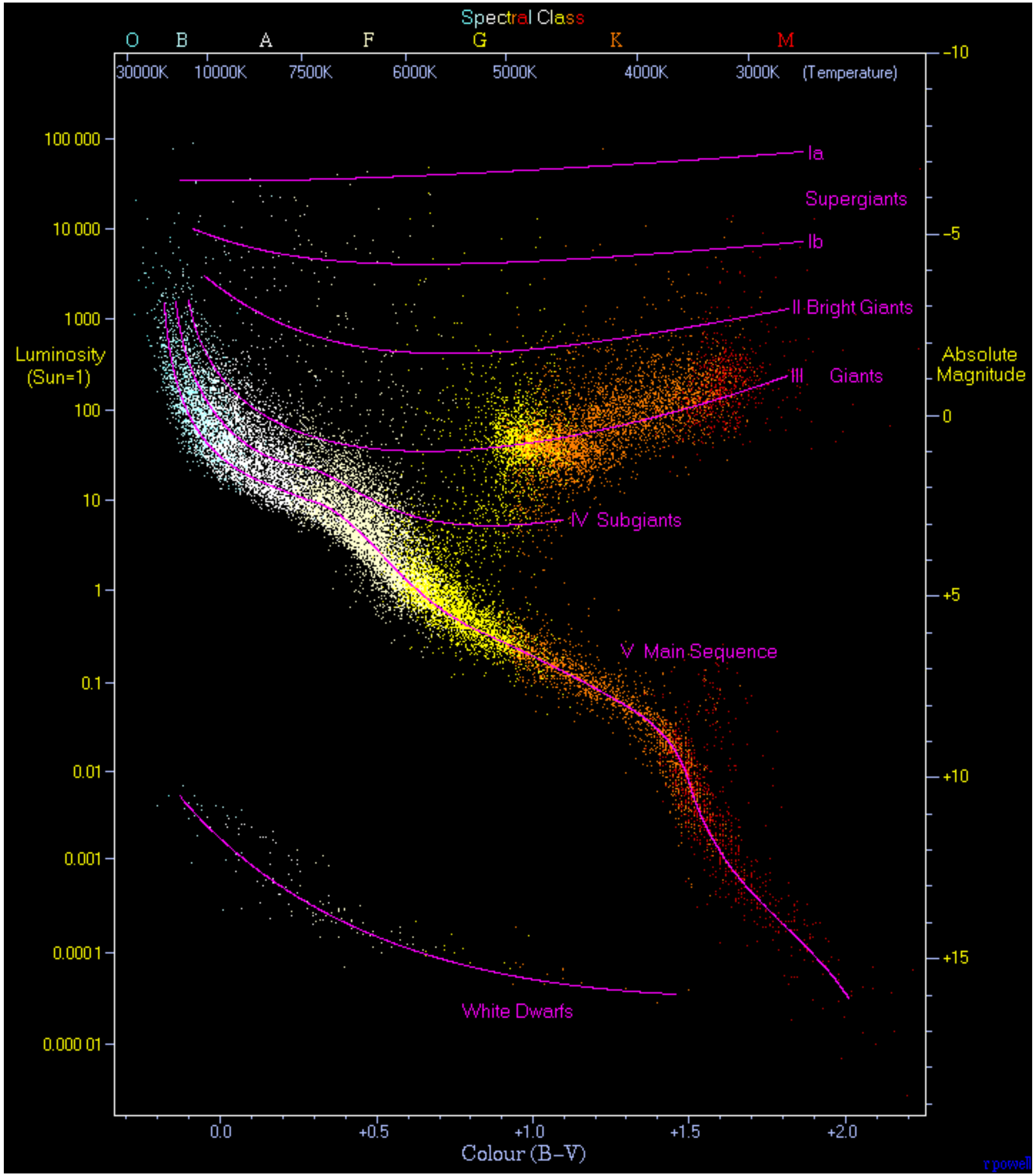
$$g \equiv \frac{G\mathcal{M}}{R^2} .$$

Chemical compositions: metallicity, Z, or iron abundance

$$\left[\frac{\text{Fe}}{\text{H}} \right] \equiv \log \left[\frac{n(\text{Fe})}{n(\text{H})} \right]_{\text{star}} - \log \left[\frac{n(\text{Fe})}{n(\text{H})} \right]_{\odot} .$$

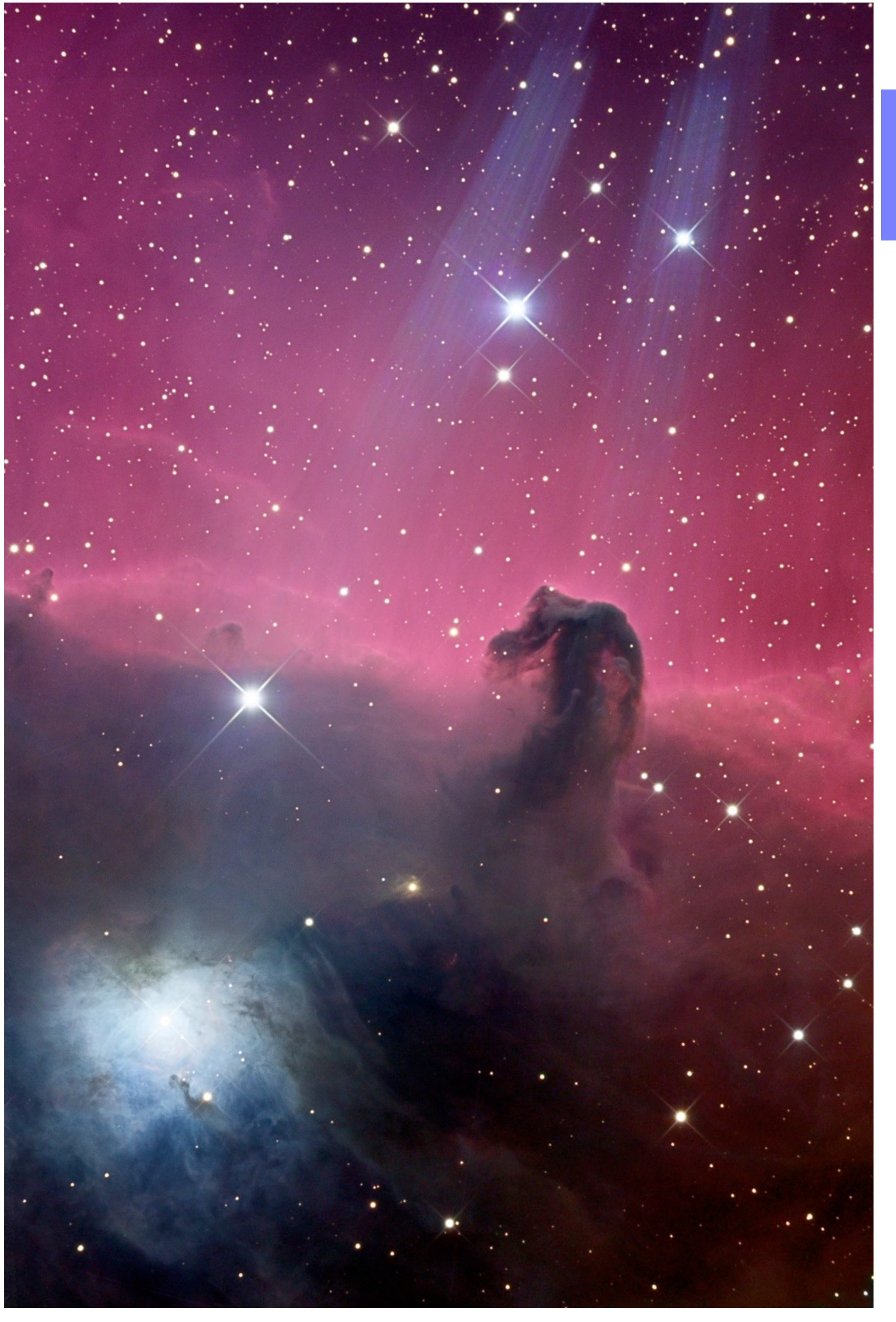
Hertzsprung-Russell Diagram

22000 stars from Hipparcos catalog and 1000 from Gliese catalog of nearby stars



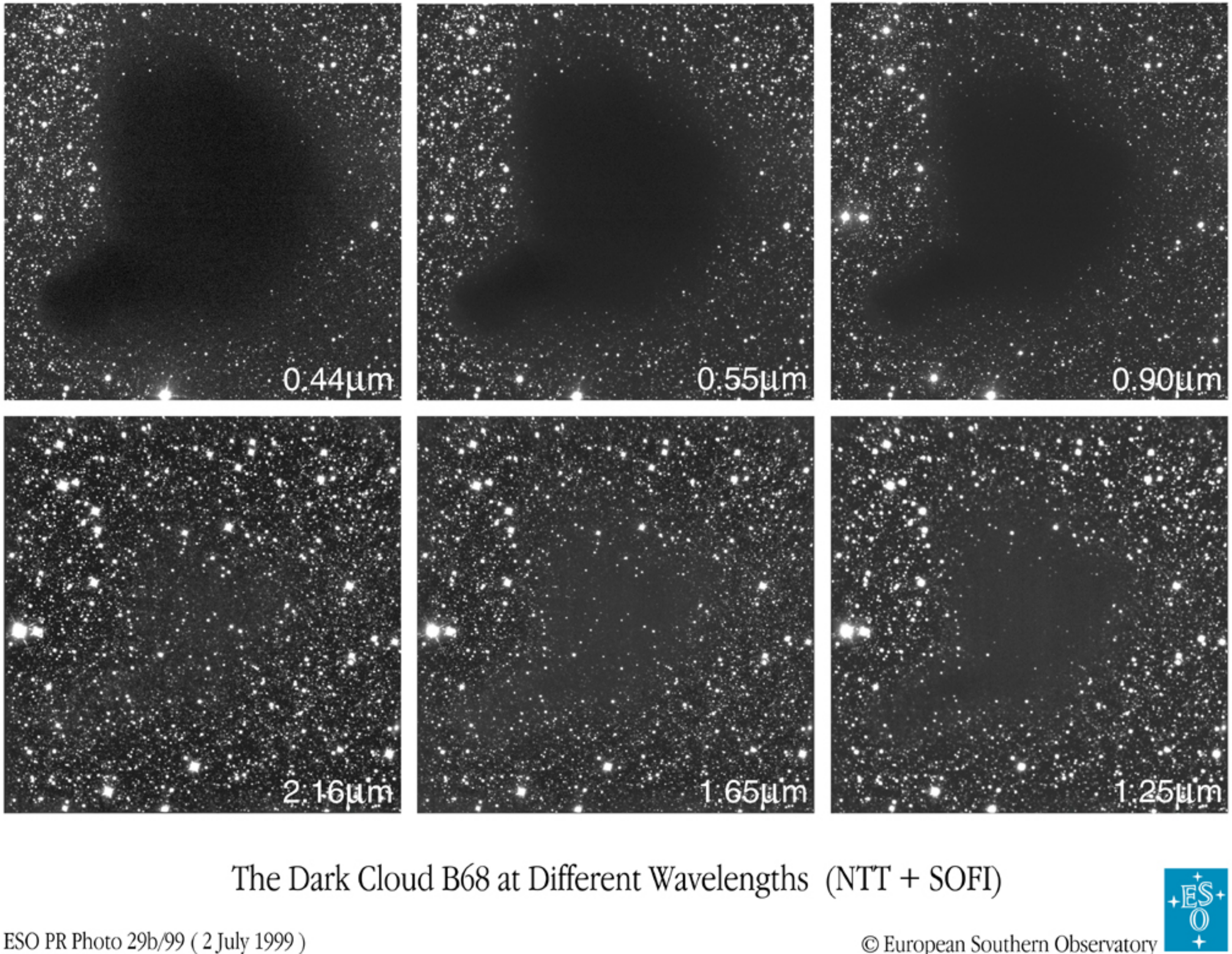
Interstellar Medium

- Display of ISM
  - Absorption: dark clouds
  - Scattering: reflection nebulae
  - Emission: emission nebulae



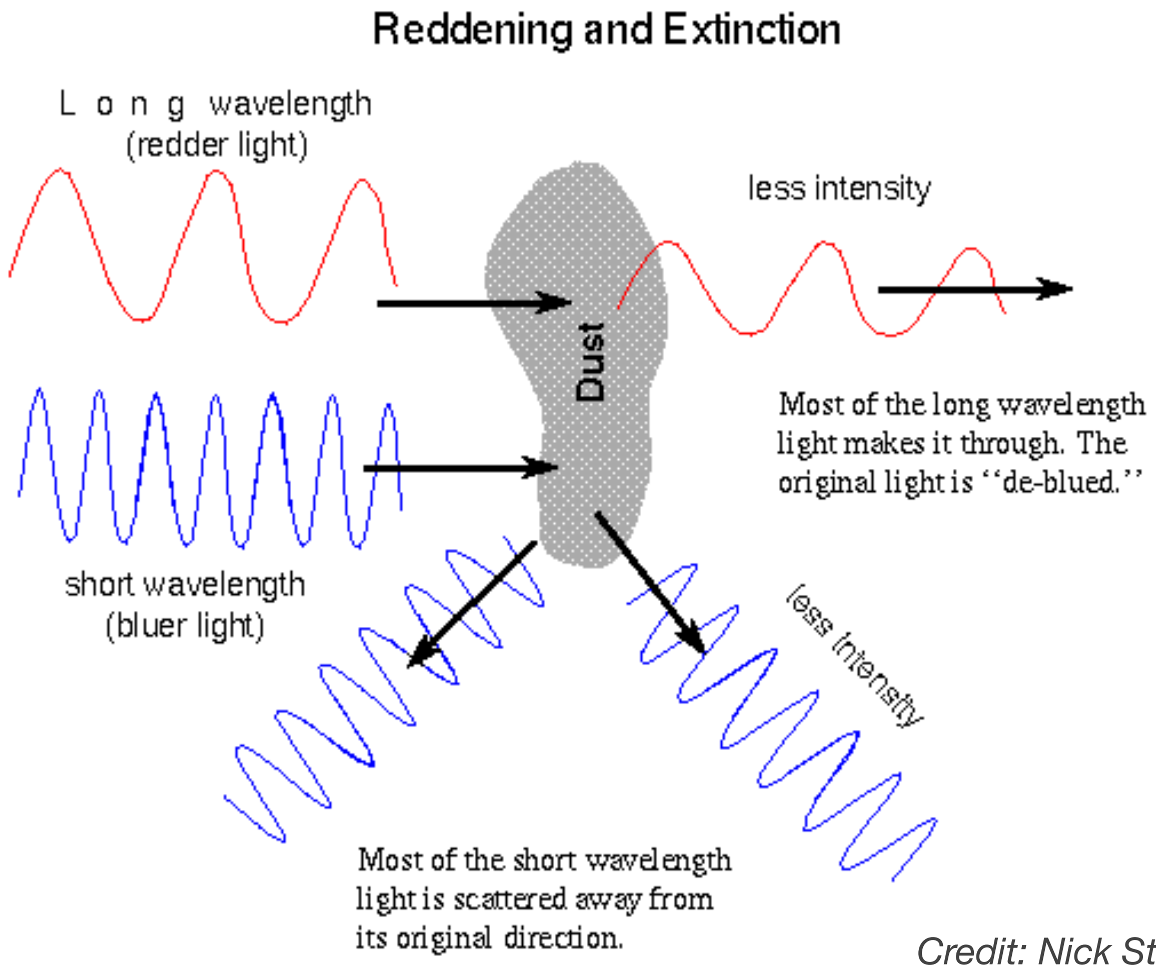
Credit: Daniel Verschatse (Antilhue Observatory)

Interstellar Extinction



Interstellar Dust

- Extinction: measure of absorption
- Reddening: measure of frequency response, i.e. colors



Credit: Nick Strobel, Astronomy Notes

Extinction & Reddening

Extinction, A_X

$$A_X \equiv m_X - m_{X,0}$$

where $m_{X,0}$ is the magnitude that would be observed in the absence of dust.

Reddening or color excess, $E(X-Y)$

$$\begin{aligned} E(X-Y) &\equiv [m_X - m_Y] - [m_{X,0} - m_{Y,0}] \\ &= A_X - A_Y \end{aligned}$$

Measured A_X **when** m_X **is known**

$$m_X = M_X + A_X + 5\log d - 5$$

Standard ISM Extinction Law

Band X	$\frac{E(X-V)}{E(B-V)}$	$\frac{A_X}{A_V}$
U	1.64	1.531
B	1	1.324
V	0	1
R	-0.78	0.748
I	-1.6	0.482
J	-2.22	0.282
H	-2.55	0.175
K	-2.74	0.112
L	-2.91	0.058
M	-3.02	0.023
N	-2.93	0.052

Extinction Curve

$\frac{A_V}{A_J R_V}$: **slope of extinction curve near the V band**

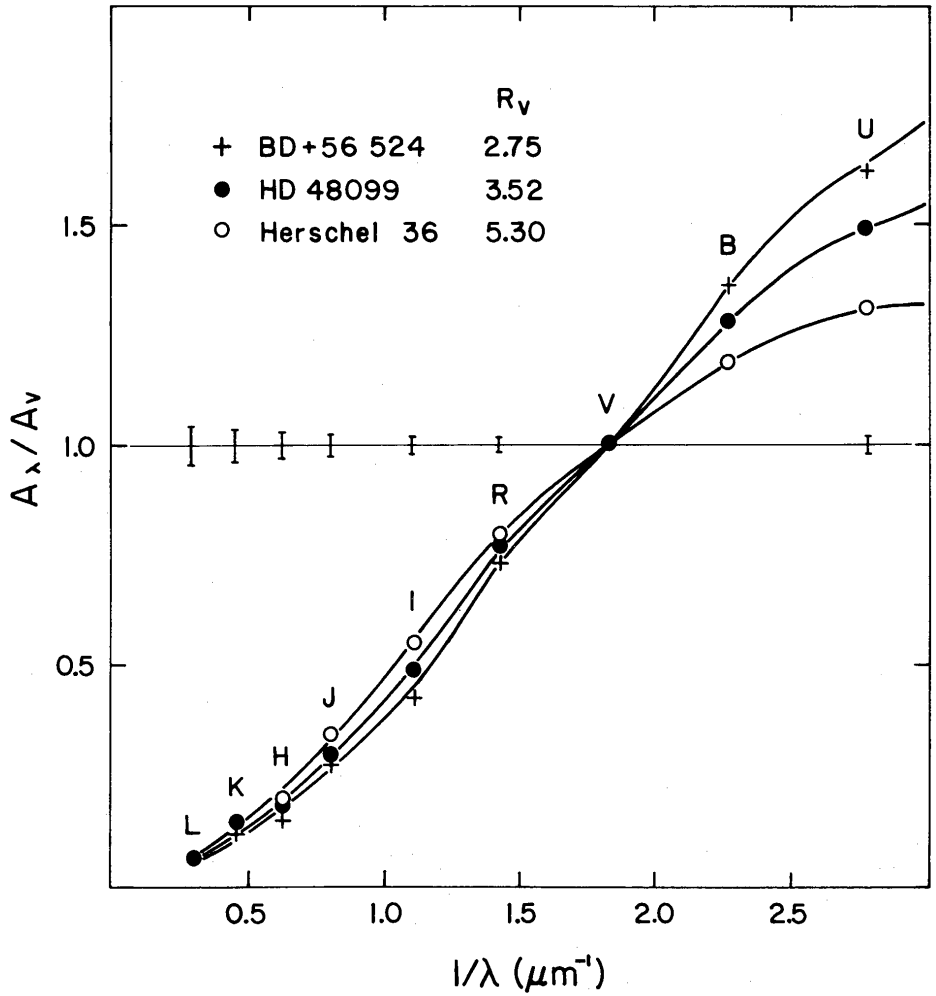
$$R_V \equiv \frac{A_V}{A_B - A_V} = \frac{A_V}{E(B-V)} \simeq 3.1$$

Gas-to-dust ratio

$$\frac{N_{\rm H}}{E(B-V)} = 5.8 \times 10^{21}\ {\rm cm^{-2}\,mag^{-1}},$$

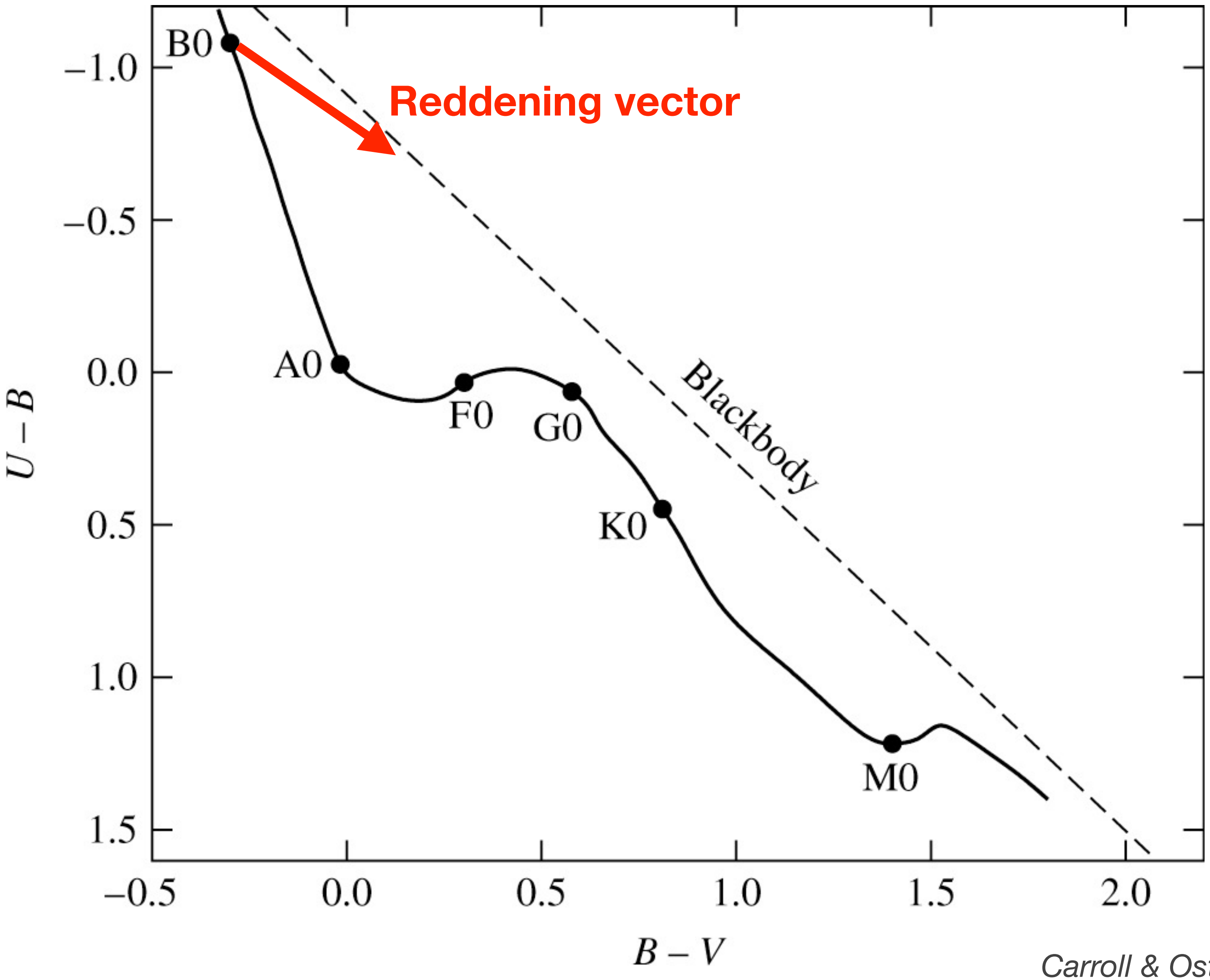
equivalent to a mass ratio of 100.

One may infer the column density $N_{\rm H}$ (cm^{-2}) by measuring the color excess $E(B-V)$ (mag).



Cardelli, Clayton, & Mathis, 1989, ApJ, 345, 245
Mathis 1990, ARA&A, 28, 37

Color-Color Diagram



Carroll & Ostlie Fig. 3.11

Reddening-Free Indices

Reddening-free indices: a photometric parameter that depends only on the spectral type of a star and is independent of the amount of reddening.

$$Q \equiv (U-B) - \frac{E(U-B)}{E(B-V)}(B-V)$$
$$\simeq (U-B) - 0.72(B-V).$$

For early-type stars of spectral types O through A0, Q determines uniquely a star's intrinsic color from photometric data alone without the need for their spectra. One finds that

$$(B-V)_0 = 0.332Q.$$

Spectral Type	Q	Spectral Type	Q
O5	-0.93	B3	-0.57
O8	-0.93	B5	-0.44
O9	-0.9	B6	-0.37
B0	-0.9	B7	-0.32
B0.5	-0.85	B8	-0.27
B1	-0.78	B9	-0.13
B2	-0.7	A0	0

Distances for Nearby Stars

- Photometric parallax
  - $d = 10^{(m-M+5)/5}$
- Trigonometric parallax
- Moving-cluster method
- Secular parallax
- Statistical parallax

Trigonometric Parallax

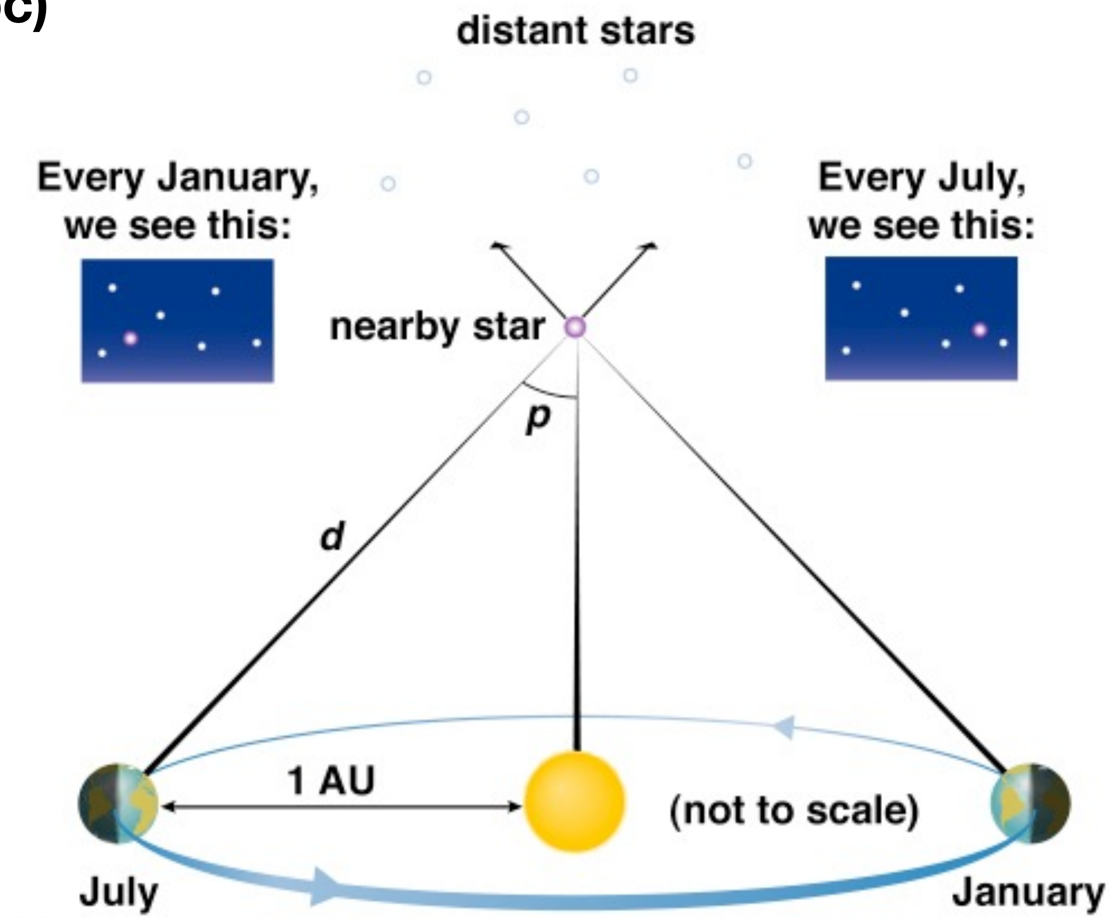
- Require stationary background references
- Hipparcos: accuracy of ~ 2 mas
- parsec (parallax-second; pc)

  = 3.09×10^{18} cm

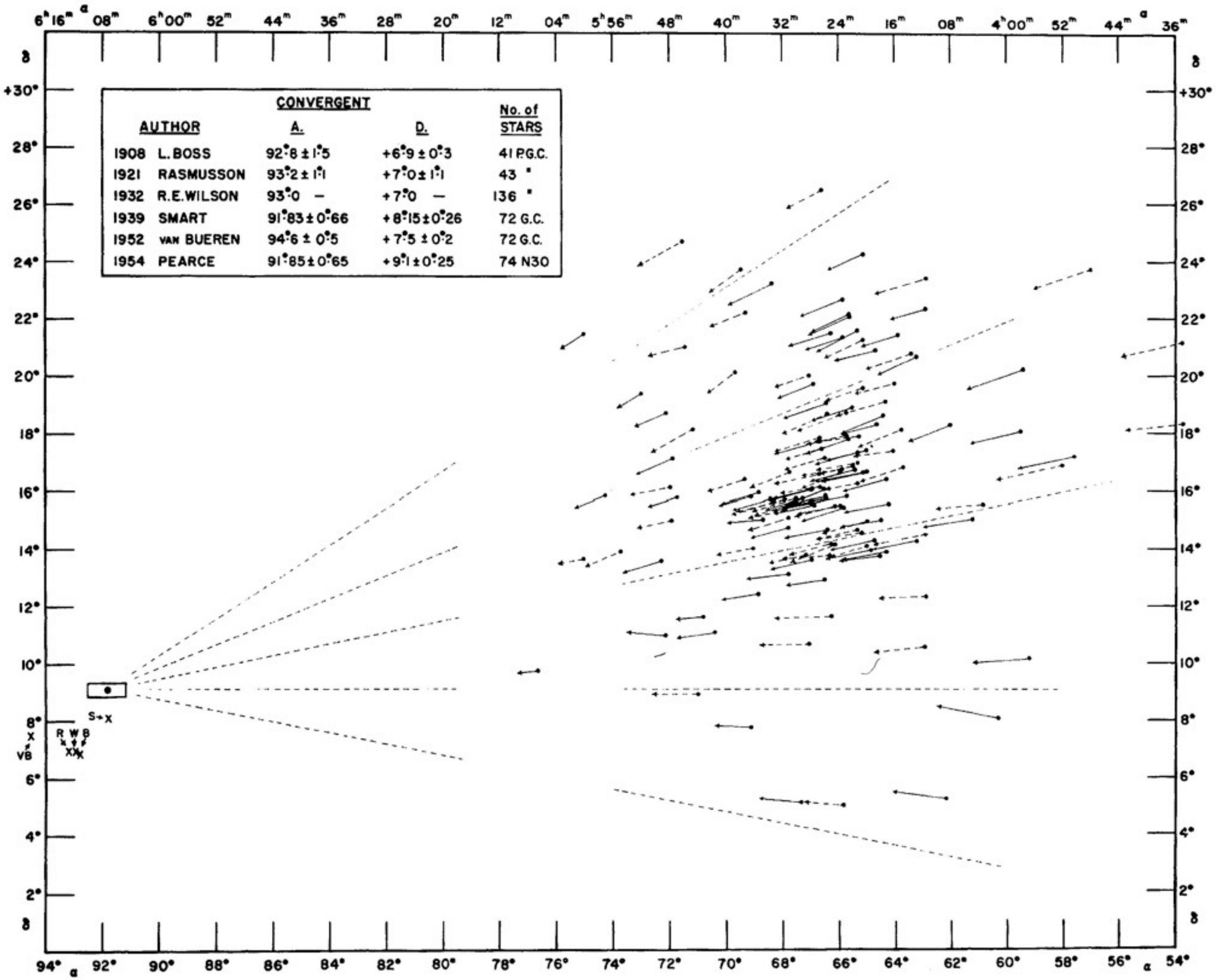
  = 3.26 light years

  = 206264.8 AU

$$\frac{d}{\mathrm{pc}} = \frac{1}{p''}$$



Moving Cluster Method I



AUTHOR		CONVERGENT A.	D.	No. of STARS
1908	L. BOSS	92°8 ± 1°5	+6°9 ± 0°3	41 P.G.C.
1921	RASMUSSON	93°2 ± 1°1	+7°0 ± 1°1	43 "
1932	R.E. WILSON	93°0 —	+7°0 —	136 "
1939	SMART	91°83 ± 0°66	+8°15 ± 0°26	72 G.C.
1952	VAN BUEREN	94°6 ± 0°5	+7°5 ± 0°2	72 G.C.
1954	PEARCE	91°85 ± 0°65	+9°1 ± 0°25	74 N30

Proper motions of Hyades members

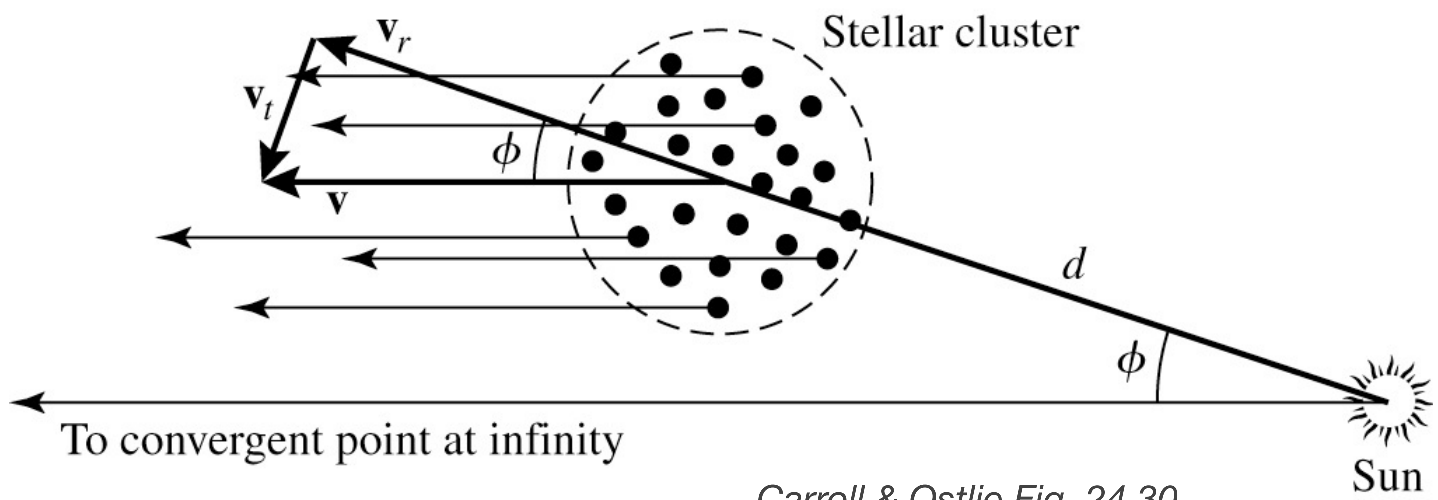
Carroll & Ostlie Fig. 24.29

Moving Cluster Method II

Determine the distance of a star cluster with identified members and the convergent point

$$\frac{d}{\mathrm{pc}} = \frac{\tan\phi}{4.74}\left(\frac{v_r}{\mathrm{km\,s^{-1}}}\right)\left(\frac{\mu}{\mathrm{mas\,yr^{-1}}}\right)^{-1}.$$

$$\begin{aligned} v_t &= v_r \tan\phi \\ &= \mu d \end{aligned}$$

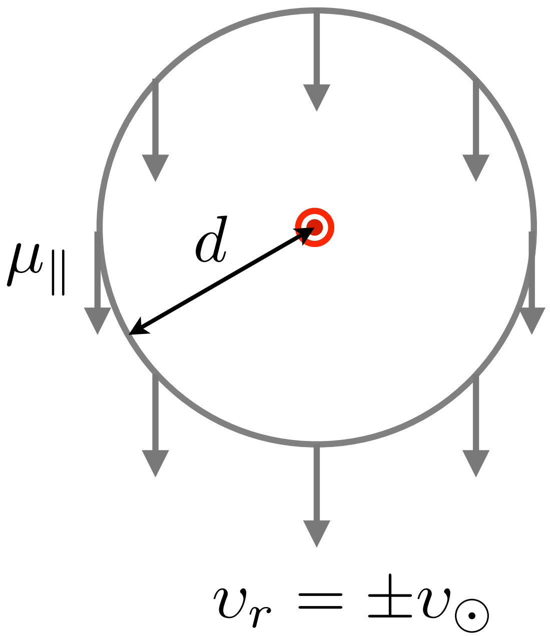


Carroll & Ostlie Fig. 24.30

Secular Parallax I

- **Utilize solar peculiar motion to extend the baseline for trigonometric parallaxes**
  - Solar motion = 13.4 km s^{-1} = 2.8 AU yr^{-1}
- **Select stars of same spectral type, same distance (apparent magnitude)**

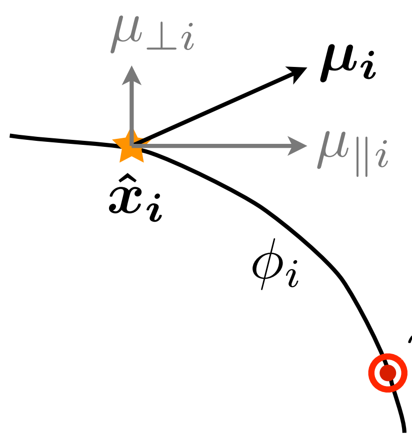
$$d = \frac{v_\odot}{\mu_{||}}$$

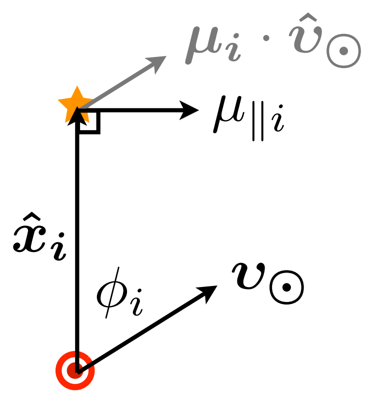


Secular Parallax II

Choose a frame in which the mean velocity of the stars of the chosen group is zero, i.e. $\sum_i \boldsymbol{v_i} = 0$. The heliocentric velocity of the $i^{\rm th}$ star is

$$\boldsymbol{u_i} \equiv \boldsymbol{v_i} - \boldsymbol{v_\odot}$$





Given the vector identity $(\boldsymbol{a} \times \boldsymbol{b}) \cdot \boldsymbol{c} = (\boldsymbol{b} \times \boldsymbol{c}) \cdot \boldsymbol{a}$, one can prove that

$$\boldsymbol{\mu_i} = \frac{(\boldsymbol{u_i} \times \boldsymbol{\hat{x}_i}) \times \boldsymbol{\hat{x}_i}}{x_i} = \frac{((\boldsymbol{v_i} - \boldsymbol{v_\odot}) \times \boldsymbol{\hat{x}_i}) \times \boldsymbol{\hat{x}_i}}{x_i}.$$

$$\mu_{\parallel i} = \frac{\boldsymbol{\mu_i} \cdot \boldsymbol{\hat{v}_\odot}}{\sin \phi_i} = \frac{(((\boldsymbol{v_i} - \boldsymbol{v_\odot}) \times \boldsymbol{\hat{x}_i}) \times \boldsymbol{\hat{x}_i}) \cdot \boldsymbol{\hat{v}_\odot}}{x_i \sin \phi_i}$$

$$= \frac{(\boldsymbol{\hat{x}_i} \times \boldsymbol{\hat{v}_\odot}) \cdot ((\boldsymbol{v_i} - \boldsymbol{v_\odot}) \times \boldsymbol{\hat{x}_i})}{x_i \sin \phi_i}.$$

Secular Parallax III

$$\mu_{\parallel i} = \frac{(\hat{\boldsymbol{x}}_{\boldsymbol{i}} \times \hat{\boldsymbol{v}}_\odot) \cdot ((\boldsymbol{v}_{\boldsymbol{i}} \times \hat{\boldsymbol{x}}_{\boldsymbol{i}}) + (\hat{\boldsymbol{x}}_{\boldsymbol{i}} \times \boldsymbol{v}_\odot))}{x_i \sin\phi_i}$$

$$\Rightarrow x_i = \frac{v_\odot |\hat{\boldsymbol{x}}_{\boldsymbol{i}} \times \hat{\boldsymbol{v}}_\odot|^2}{\mu_{\parallel i} \sin\phi_i} - \frac{(\hat{\boldsymbol{x}}_{\boldsymbol{i}} \times \hat{\boldsymbol{v}}_\odot) \cdot (\hat{\boldsymbol{x}}_{\boldsymbol{i}} \times \boldsymbol{v}_{\boldsymbol{i}})}{\mu_{\parallel i} \sin\phi_i}$$

$$= v_\odot \frac{\sin^2\phi_i}{\mu_{\parallel i} \sin\phi_i} - \underbrace{\frac{(\hat{\boldsymbol{x}}_{\boldsymbol{i}} \times \hat{\boldsymbol{v}}_\odot) \cdot (\hat{\boldsymbol{x}}_{\boldsymbol{i}} \times \boldsymbol{v}_{\boldsymbol{i}})}{\mu_{\parallel i} \sin\phi_i}}_{=0}.$$

Taking average over all N stars, we find that the second term vanishes since $\sum_i \boldsymbol{v}_{\boldsymbol{i}} = \boldsymbol{0}$ by our choice of reference frame. Given $\langle p \rangle \equiv 1/x_i$, we obtain

$$\boxed{\langle p \rangle = \frac{\langle \mu_{\parallel i} \sin\phi_i \rangle}{v_\odot \langle \sin^2\phi_i \rangle}}$$

Statistical Parallax

Statistical parallaxes are similar to secular parallaxes but use the perpendicular component of the proper motions. An additional assumption is introduced that the velocities $\boldsymbol{v}_i$ are isotropically distributed. the observed radial velocity of the i^{th} star is

$$\begin{aligned} u_{ri} &= \hat{\boldsymbol{x}}_i \cdot (\boldsymbol{v}_i - \boldsymbol{v}_\odot) \\ &= \hat{\boldsymbol{x}}_i \cdot \boldsymbol{v}_i - v_\odot \cos\phi_i . \end{aligned}$$

The component of $\boldsymbol{v}_i$ perpendicular to the plane congaing the Sun, the star, and $\boldsymbol{v}_\odot$ is $x_i \boldsymbol{\mu}_{\perp i}$, and by the hypothesis that the mean magnitude of any component of $\boldsymbol{v}_i$ is the same, we have

$$\langle |\hat{\boldsymbol{x}}_{\boldsymbol{i}} \cdot \boldsymbol{v}_{\boldsymbol{i}}| \rangle = \langle |x_i \boldsymbol{\mu}_{\perp \boldsymbol{i}}| \rangle .$$

Therefore, the statistical parallax is

$$\boxed{\langle p \rangle = \frac{\langle |\mu_{\perp i}| \rangle}{\langle u_{ri} + v_\odot \cos\phi_i \rangle}}$$

Note that isotropic assumption is not always valid. A more sophisticated analysis involves the ellipsoidal shape of the random velocity distribution is also available.

Summary of Methods

- **Supergiants**
  - clusters
- **O-A stars**
  - clusters, secular and statistical parallaxes
- **F-M dwarfs**
  - trigonometric parallaxes, moving cluster method
- **F-M giants**
  - moving cluster method, clusters, secular and statistical parallaxes
- **White dwarfs**
  - trigonometric parallaxes, binaries, clusters

Stellar Luminosity Function

- $\mathrm{d}N$ = **number of stars with absolute magnitude** $(M+\mathrm{d}M, M)$ **in the volume** $\mathrm{d}^3\boldsymbol{x}$ **around the point** $\boldsymbol{x}$

$$\mathrm{d}N = \Phi(M, \boldsymbol{x})\, \mathrm{d}M\, \mathrm{d}^3\boldsymbol{x}$$

- **General luminosity function** $\Phi(M)$
  - **Irrespective to spectral types**

$$\mathrm{d}N = \Phi(M)\mathrm{d}M \;\; n(\boldsymbol{x})\mathrm{d}^3\boldsymbol{x}$$

relative fractions of stars with different luminosities

number density of stars around $\boldsymbol{x}$

Biases in Counting

- **Occurs in a survey of fixed solid angle, in which the volume studied increases with distance**
- **Malmquist bias** (Malmquist 1922, 1936)
  - **The mean absolute magnitude of observed sample is brighter than the mean absolute magnitude of the population**
  - **Caused by magnitude-limited survey, i.e. brighter stars can be seen farther**
- **Lutz-Kelker bias** (Lutz & Kelker 1973, PASP, 85, 573)
  - **Observed parallax to be on average higher than its true value**
    - ➫ Underestimate of distance
    - ➫ Underestimate of intrinsic luminosity

Malmquist Bias

A bias caused by a **magnitude-limited** sample, i.e. usual observations, comparing to a **volume-limited** sample, i.e. intrinsic populations. The objects in a survey that have a given apparent magnitude, m, will, on the average, have a higher luminosity than the mean luminosity of the population as a whole. In short, the apparent magnitude and the dispersion are

$$\boxed{\langle M \rangle_m - M_0 = -\sigma^2 \frac{\mathrm{d} \ln A}{\mathrm{d}m},} \quad \text{and} \quad \boxed{\sigma_m^2 - \sigma^2 = \sigma^4 \frac{\mathrm{d}^2 \ln A}{\mathrm{d}m^2}.}$$

$$A(m) \equiv \frac{\mathrm{d}N}{\mathrm{d}m}, \qquad \text{star-count function}$$

$$\Phi(M) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(M - M_0)^2}{2\sigma^2} \right),$$

$$\sigma_m^2 \equiv \langle M^2 \rangle_m - \langle M \rangle_m^2,$$

where M_0 and σ are the mean absolute magnitude and the dispersion in absolute magnitude of a **volume-limited** sample.

Lutz-Kelker Bias I

For a survey of fixed solid angle, the volume studied increases with distance and causes an observed trigonometric parallax to be on average higher than its true value, i.e. an underestimate of distance and hence the luminosity. Let $P(p|p')\,\mathrm{d}p$ be the probability that the true parallax of a given star lies in $(p, p + \mathrm{d}p)$ given its measured parallax is p'. The probability is given by

$$
\begin{aligned}
P(p|p') &\propto P(p'|p)\,P(p) \\
&\propto P(p'|p)\Phi(M)\,n(s)\,s^2\,\left|\frac{\partial s}{\partial p}\right|_m , \\
&\propto \Phi(M)\,p^{-4}\,\exp\left[-\frac{(p'-p)^2}{2\sigma_p^2}\right],
\end{aligned}
$$

where $s = 1/p$ and $M = m + 5\log(p/10)$. Note that Lutz-Kelker bias is strongly biased toward small values of p and moderated by the luminosity function, $\Phi(M)$.

Lutz-Kelker Bias II

Mon. Not. R. Astron. Soc. **338,** 891–902 (2003)

Is there really a Lutz–Kelker bias? Reconsidering calibration with trigonometric parallaxes

Haywood Smith, Jr★

Department of Astronomy, University of Florida, Gainesville, Florida USA 32611

ABSTRACT

In the recent literature there are indications of some confusion regarding the Lutz–Kelker bias: whether or not it exists, and if so, what it is and when it should be corrected. Here we carefully reexamine Lutz & Kelker's original work to understand what they actually did, and then look at their later papers and some other works on the subject. There is, properly speaking, no universal Lutz–Kelker bias of individual parallaxes. There is a bias for stars that are members of samples which is different from, but often has the same form as and is given the name of, the Lutz–Kelker bias. The overall bias for samples selected according to relative parallax error is sometimes given the name of Lutz–Kelker; in fact it is, or is very nearly the same as, that discussed by Trumpler and Weaver. The Lutz–Kelker corrections can, under certain conditions, be used to counter that bias. The Lutz–Kelker correction applied for an isolated star (independent of sample properties) is an incomplete refinement of the estimate of absolute magnitude calculated directly from the parallax, not a correction for bias.

We reconsider the more general problem of calibration using only trigonometric parallaxes, and examine some of the maximum likelihood methods proposed for its solution. Of these, several are based on linear approximation and are therefore of limited validity. Three exact methods are all based on essentially the same form of the likelihood but are implemented in different ways. One of these is statistically flawed, as was originally pointed out by Jung. We test the other two using synthetic samples, compare their performance, and discuss their application. We also apply one (grid method) to the Feast–Catchpole high weight sample of *Hipparcos* Cepheid parallaxes as a test. The grid method is to be preferred over the approximate ones because it does not have a limited range of validity, should not require any a posteriori correction, and provides a more complete picture of the uncertainties, in the form of a contour diagram of log(likelihood).

Lutz-Kelker Bias III

MNRASL **444,** L6–L10 (2014) doi:10.1093/mnrasl/slu103

The Lutz–Kelker paradox

Charles Francis★
25 Elphinstone Rd, Hastings, TN34 2EG, UK

Accepted 2014 June 25. Received 2014 June 20; in original form 2014 April 7

ABSTRACT

The Lutz–Kelker correction is intended to give an unbiased estimate for stellar parallaxes and magnitudes, but it is shown explicitly that it does not. This paradox results from the application of an argument about sample statistics to the treatment of individual stars, and involves the erroneous use of a frequency distribution in the manner of a probability density function considered as a Bayesian prior. It is shown that the Bayesian probability distribution for true parallax given the observed parallax of a selected star is independent of the distribution of other stars. Consequently, the Lutz–Kelker correction should not be used for individual stars. This result has important implications for the RR Lyrae scale and for the interpretation of results from *Gaia* and *Hipparcos*. The Lutz–Kelker correction is a poor treatment of the Trumpler–Weaver bias which affects parallax limited samples. A true correction is calculated using numerical integration and confirmed by a Monte Carlo method.

Key words: astrometry – parallaxes.