Galactic Astronomy, Spring 2017

PROBLEM SET III

Deadline: 5PM OF WEDNESDAY, MAY 3, 2017

For the following problems, assume the Sun rotates about the Galactic center with a circular speed Θ_0 at a distance of $R = R_0$. Do **not** apply Oort constants since approximation is not necessary here.

1. **Galactic longitude-veloity (ℓ-v) diagram (40%+10%).** Imaging that there is a ring of atomic hydrogen (H I) gas at radius R from the center of the Milky Way. For each of the following cases, derive an expression for the *observed radial velocity*, v_r, of this ring of gas as a function of R and Galactic longitude, ℓ.

   (a) The ring of gas is rotating with a velocity $V_c = \Theta_0$. Plot your results, including cases for both $R < R_0$ and $R > R_0$, on a longitude-velocity diagram. (15%)

   (b) The ring of gas is rotating with a velocity $V_c = \Theta_0$ at the solar circle, that is, $R = R_0$. (5%)

   (c) The ring of gas is expanding with a velocity V_e but no rotation, i.e. $V_c = 0$. *You may ignore the solar motion (observer's velocity) for this part to better see the behavior of an expanding ring.* Plot your results on a longitude-velocity diagram. (10%)

   (d) The ring of gas is at $R < R_0$ rotating with a velocity V_c and expanding with a velocity V_e. *Please include the solar motion for this part.* (10%)

   (e) (Bonus) Describe the results of (d) on a longitude-velocity diagram. (10%)

2. **Distance ambiguity (50%).** In class, we briefly touched the distance ambiguity problem: one can always find at least two locations in the inner part ($R < R_0$) of the Milky Way with same radical velocity, v_r.

   (a) Prove that a ring of gas at $R < R_0$ will always have two locations with same radial velocity, v_r, with the exception of the tangent point. (10%)

   (b) Use the law of cosines, prove the geometry $d = R_0 \cos \ell \pm (R^2 - R_0^2 \sin^2 \ell)^{1/2}$. (10%)

(c) Prove that the maximum radial velocity, $v_r^{\max}$, at longitude ℓ is given by the tangent point, at which $d = R_0 \cos\ell$ and $v_r^{\max} = \Theta - \Theta_0 \sin\ell$. (10%)

(d) Draw a circle to represent the solar circle, label the Sun and its motion, and place a line through $\ell = 0°$ and $180°$. Such a cartoon basically divides the Milky Way into four different regions with two sets of criterions: $R < R_0$ or $R > R_0$, and $0° < \ell < 180°$ or $-180° < \ell < 0°$. Describe signs of radial velocity, positive and red-shifted (+), or negative and blue-shifted (−), in these four regions on the graph. Explain clearly the reason for your answers. (10%)

(e) Explain why the outer part ($R > R_0$) of the Galaxy does not suffer from the distance ambiguity the way the inner part does. (10%)