Galactic Astronomy, Spring 2017

Problem Set V

Deadline: 5PM of Wednesday, May 31, 2017

1. **Tully-Fischer Law (30%).** The use of the virial theorem, together with the hypothesis of a universal mass distribution of a constant central surface density $\Sigma_0 =$ constant in spiral galaxies, allows us to give a physical interpretation of the Tully-Fisher relation $L_{\mathrm{H}} \propto \sigma_v^4$ (or $M \propto \sigma_v^4$ if $M/L_{\mathrm{H}} =$ constant).

   (a) (10%) The density in spiral galaxies can be written as

   $$\rho(r) = \rho_0 \, f\left(\frac{R}{R_0}\right) = \rho_0 \, f(x).$$

   Show that the surface density can therefore be written as

   $$\Sigma(R) = \Sigma_0 \, g\left(\frac{R}{R_0}\right) = \Sigma_0 \, g(x),$$

   and likewise the rotation curve as

   $$v_c(R) = v_{c,\mathrm{max}} \, h(x),$$

   where $f(x)$, $g(x)$, and $h(x)$ are universal functions.

   (b) (10%) Express the total kinetic energy K as a function of the parameters Σ_0, R_0, and $v_{c,\mathrm{max}}$ and a numerical factor dependent only on the universal functions.

   (c) (10%) Write down the virial theorem. Furthermore, integrate the surface density $\Sigma(R)$ to obtain the total mass M. Then show that

   $$M \propto \frac{v_{c,\mathrm{max}}^4}{\Sigma_0}.$$

2. **Galactic-Disk models (25%).** The disk of a spiral galaxy is considered infinitely thin, and a model of the surface density $\Sigma(R)$ is proposed to have the form

   $$\Sigma(R) = M \, \frac{a}{2\pi} \, (R^2 + a^2)^{-3/2},$$

   where M is the total mass of the disk and a is a scale typical for the sizes of galactic disks. The density can be written as

   $$\rho(R, z) = \Sigma(R)\delta(z).$$

Let's calculate the gravitational potential $\Phi(R, z)$ and the rotation velocity $v(R)$ at an arbitrary point in the disk.

(a) (15%) Write down the Poisson equation in cylindrical coordinates and find $\Phi(R, z)$ in the form

$$\Phi(R, z) \simeq [R^2 + (a + |z|)^2]^n.$$

What is the value of the power n for which the equation is satisfied.

(b) (10%) Find the rotation curve of this self-gravitating disk.