Galactic Astronomy, Spring 2017

Problem Set VI

Deadline: 5PM of Wednesday, June 14, 2017

1. **Isothermal stellar systems (20%).** Consider a distribution function, $f(E) \propto e^{-\beta E}$, for a spherical, isothermal distribution of stars.

   (a) (10%) Solve the Poisson equation exactly and calculate the potential $\Phi(r)$.

   (b) (10%) Determine the density $\rho(r)$ at every point.

2. **Epicyclic motions (40%).** In the epicycle approximation, we have shown in class that a star moves on an ellipse (epicycle) around the guiding center (epicenter) that traces a circular orbit around the galactic center with an angular velocity $\Omega_g = \upsilon_c(R_g)/R_g$. *Hint:* You are encouraged to read on Sect. 3.2.3 in *Galactic Dynamics* by Binney & Tremaine.

   (a) (10%) Show that the lengths of the semi-axes of the epicycle are in the ratio

   $$\frac{X}{Y} = \frac{\kappa}{2\,\Omega_g},$$

   where X and Y are the maximum position offsets in the radial and azimuthal directions, respectively. In general, $\kappa < 2\,\Omega_g$, and the epicycle is elongated along the tangential direction. *Hint:* You may express the solution of the offset in the radial direction as $x(t) = X\cos(\kappa t + \phi_0)$.

   (b) (10%) Consider the motion of a star that moves on an epicyclic orbit, as viewed by an astronomer who sits at the epicenter of the star's orbit. At different times in the orbit, the astronomer's proper motion measurements yield the maximum values κX and κY, which in turn yield important information about the galactic potential. However, the epicycle period is much longer than an astronomer's lifetime, so he should try to find the result more quickly by averaging the results from many stars whose orbits are differ only in their epicycle phases, ϕ_0, to obtain the radial velocity dispersion, σ_r, and the tangential velocity dispersion, σ_ϕ, with

   $$\begin{aligned} \sigma_r^2 &\equiv \overline{\upsilon_r^2} = \overline{\dot{x}^2} \\ \sigma_\phi^2 &\equiv \overline{\upsilon_\phi - \upsilon_c(R_g)} = \overline{\dot{y}^2}. \end{aligned}$$

Show that σ_r and σ_ϕ have the ratio

$$\frac{\sigma_r}{\sigma_\phi} = \frac{\kappa}{2\,\Omega_g}.$$

(c) (20%) However, the above procedure is not practical at all since in general we do not know that location of the epicenter of any given star. Instead, we can only measure v_r and $v_\phi(R) - v_c(R)$ for a group of stars, each of which has its own epicentric radius R_g, as they pass near our sampled radius R (for example, Fig. 1). Again, show that the ratio between radial velocity dispersion, σ_r, and tangential velocity dispersion, σ_ϕ, have the inverse ratio

$$\frac{\sigma_r}{\sigma_\phi} = \frac{2\,\Omega_g}{\kappa}.$$

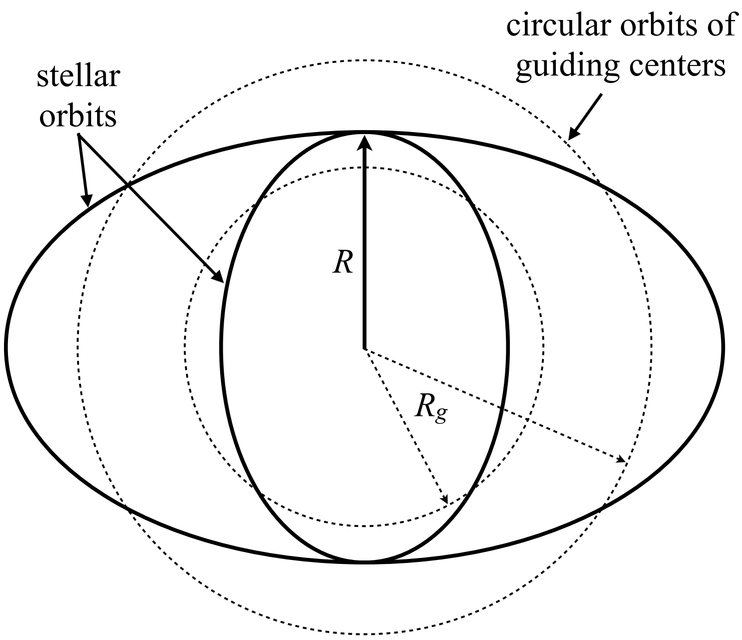

Figure 1: Stars with orbits that pass near R with different epicentric radii R_g.