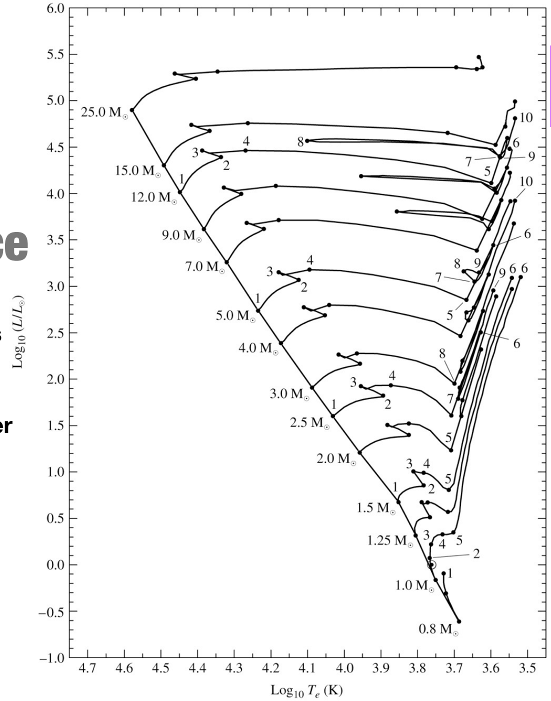
Chapter 13
Main Sequence & Post-Main-Sequence Stellar Evolution

Evolution on main sequence
Late stages of stellar evolution
Stellar clusters and stellar populations

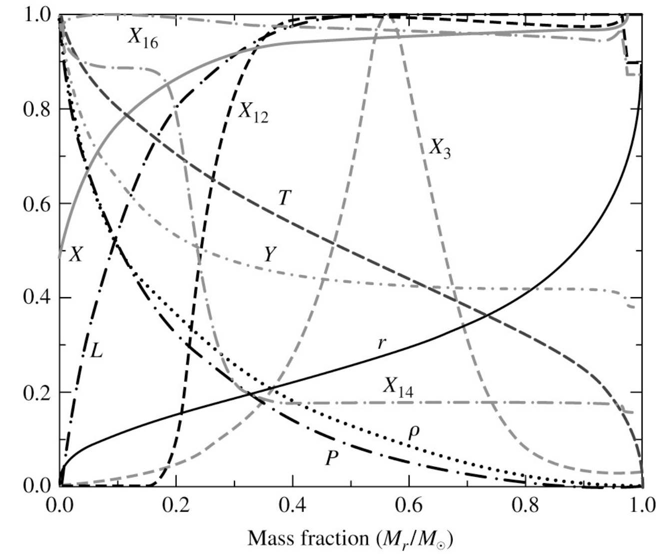
Evolution on Main Sequence

- Chemical compositions slowly changed by nuclear reactions
- Evolving toward brighter and redder
  - 4H → He reducing the number of particles, causing core contraction and energy increase
  - $\varepsilon_{pp} \propto \rho X^2 T_6^4$
- The more massive, the faster the evolution



Interior Structure of the Sun

Current age
~ 4.57 Gyr



He Core + H Shell Burning

- **Isothermal He core (near point 3): *Lr* ≃ 0 ⇒ d*T*/d*r* ≃ 0**

Since $\epsilon = 0$ in the He core,

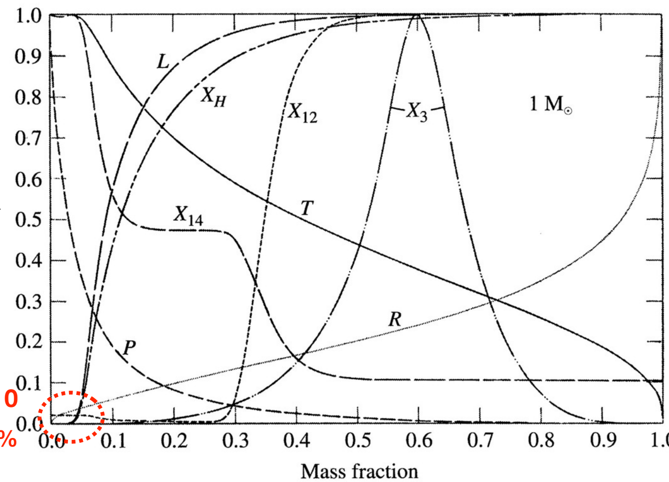
$$\frac{\mathrm{d}L_r}{\mathrm{d}r} = 4\pi r^2 \rho \epsilon = 0,$$

one obtains

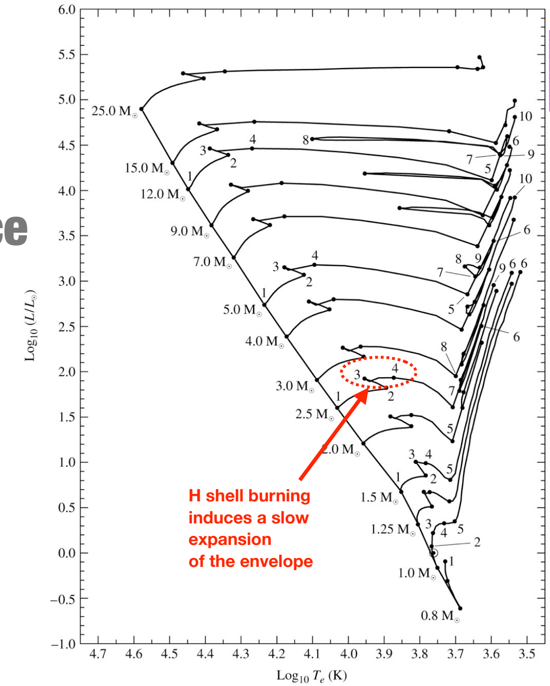
$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{3}{4ac}\frac{\bar{\kappa}\rho}{T^3}\frac{L_r}{4\pi r^2} = 0.$$

Hence the core is isothermal.

Lr ≃ 0
inner 3%



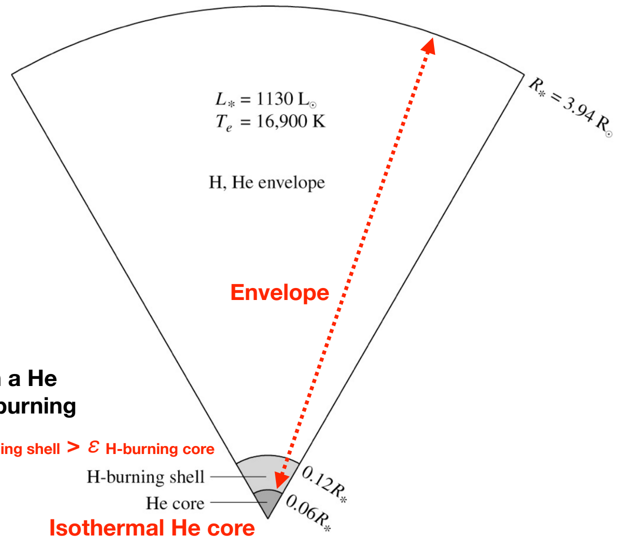
Evolution on Main Sequence



Shell Burning Structures

- $5\ M_\odot$ star with a He core and a H-burning shell

$\varepsilon_{\text{H-burning shell}} > \varepsilon_{\text{H-burning core}}$



Core Mass Limit

The *maximum fraction* of a star's mass that can exist in an *isothermal core* and still support the overlying layers is given by the **Schönberg-Chandrasekhar limit**

$$\boxed{\left(\frac{M_{\rm ic}}{M}\right)_{\rm SC} \simeq 0.37\left(\frac{\mu_{\rm env}}{\mu_{\rm ic}}\right)^2,}$$

where $\mu_{\rm env}$ and $\mu_{\rm ic}$ are the mean molecular weights of the overlying envelopes and the isothermal core, respectively.

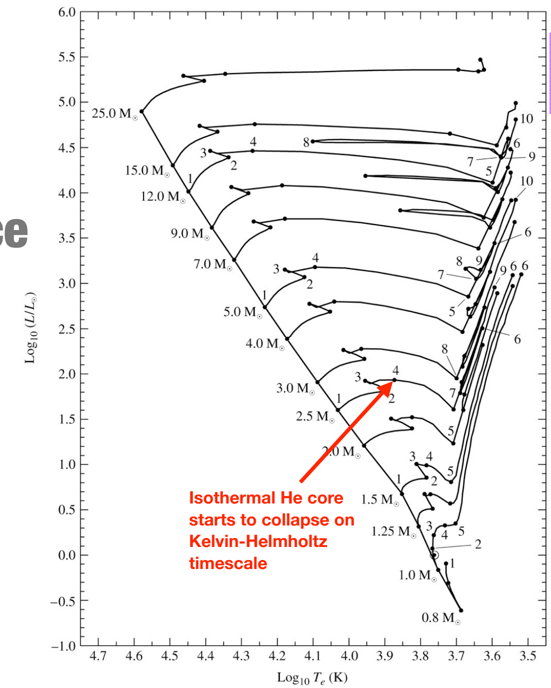
When the gas density is sufficiently high and **electron degeneracy** happens, an isothermal core may exceed the Schönberg-Chandrasekhar limit. The pressure of a completely degenerate, non-relativistic electron gas is given by

$$P_e = K\rho^{5/3}.$$

Note that P_e is independent of the temperature, T. Stars with $M \lesssim 1.5M_\odot$ become degenerate before the Schönberg-Chandrasekhar limit is reached. This in turns causes the subsequent **He core flash**.

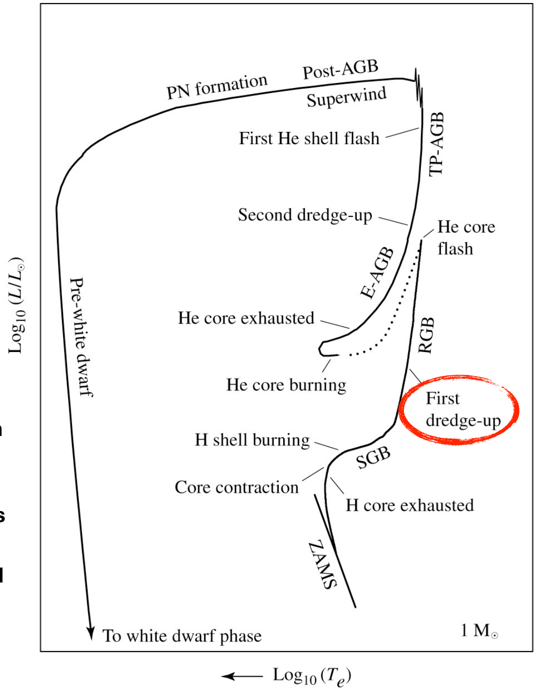
Evolution on Main Sequence

- **When $M_{\rm ic} > M_{\rm sc}$, core collapses on a Kelvin-Helmholtz timescale, and the star evolves rapidly**
- **For $M < 1.2\ M_\odot$, the collapse of the isothermal He core is the end of their MS phase**



Evolution Off the Main Sequence

- **First dredge-up due to an increase of photospheric opacity from the H^- ion**
- **Convective zone deepens into the star**
- **Transport C inward and N outward**



Isothermal Core Mass Limit (I)

The evolution phase of an isothermal He core ends when the mass of the core becomes too great and the core is no longer capable of supporting the material above it. The core then collapses on a Kelvin-Helmholtz timescale.

A "not-so-straightfoward" and "not-so-accurate" derivation of the Schönberg-Chandrasekhar limit: Rewriting the equation of hydrostatic equilibrium with the interior mass

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM_r\rho}{r^2} \quad \text{and} \quad \frac{\mathrm{d}M_r}{\mathrm{d}r} = 4\pi r^2\rho$$

$$\Rightarrow \quad \frac{\mathrm{d}P}{\mathrm{d}M_r} = -\frac{GM_r}{4\pi r^4}$$

$$4\pi r^3\frac{\mathrm{d}P}{\mathrm{d}M_r} = -\frac{GM_r}{r}.$$

Replace the LHS with

$$4\pi r^3\frac{\mathrm{d}P}{\mathrm{d}M_r} = \frac{\mathrm{d}(4\pi r^3 P)}{\mathrm{d}M_r} - 12\pi r^2 P\frac{\mathrm{d}r}{\mathrm{d}M_r} = \frac{\mathrm{d}(4\pi r^3 P)}{\mathrm{d}M_r} - \frac{3P}{\rho}$$

Isothermal Core Mass Limit (II)

Substituting and integrating over the mass of the isothermal core, $M_{\rm ic}$, we have

$$\int_0^{M_{\rm ic}} \frac{\mathrm{d}(4\pi r^3 P)}{\mathrm{d}M_r}\mathrm{d}M_r - \int_0^{M_{\rm ic}} \frac{3P}{\rho}\mathrm{d}M_r = -\int_0^{M_{\rm ic}} \frac{GM_r}{r}\mathrm{d}M_r.$$

Let's evaluate each term separately:

$$\int_0^{M_{\rm ic}} \frac{\mathrm{d}(4\pi r^3 P)}{\mathrm{d}M_r}\mathrm{d}M_r = 4\pi\, R_{\rm ic}^3\, P_{\rm ic}$$

$$\int_0^{M_{\rm ic}} \frac{3P}{\rho}\mathrm{d}M_r = \frac{3M_{\rm ic}kT_{\rm ic}}{\mu_{\rm ic}m_H} = 3N_{\rm ic}\, k\, T_{\rm ic} = 2K_{\rm ic}$$

$$-\int_0^{M_{\rm ic}} \frac{GM_r}{r}\mathrm{d}M_r = U_{\rm ic}$$

Therefore, we find a generalized form of the virial theorem for stellar interiors in hydrostatic equilibrium

$$\boxed{4\pi\, R_{\rm ic}^3\, P_{\rm ic} - 2K_{\rm ic} = U_{\rm ic}.}$$

Isothermal Core Mass Limit (III)

The gravitational energy and internal thermal energy of the core may be approximated as

$$U_{\rm ic} \sim -\frac{3}{5}\frac{GM_{\rm ic}^2}{R_{\rm ic}} \quad \text{and} \quad K_{\rm ic} = \frac{3M_{\rm ic}\, kT_{\rm ic}}{2\mu_{\rm ic}\, m_{\rm H}}.$$

Solving for the pressure at the surface of the isothermal core, we have

$$P_{\rm ic} = \frac{3}{4\pi R_{\rm ic}^3}\left(\frac{M_{\rm ic}\, kT_{\rm ic}}{\mu_{\rm ic}\, m_{\rm H}} - \frac{1}{5}\frac{GM_{\rm ic}^2}{R_{\rm ic}}\right).$$

Taking $\mathrm{d}P/\mathrm{d}M_{\rm ic} = 0$, we can calculate the maximum value of surface pressure, $P_{\rm ic,max}$, that can be produced by an isothermal core with the radius

$$R_{\rm ic} = \frac{2}{5}\frac{GM_{\rm ic}\,\mu_{\rm ic}\, m_{\rm H}}{kT_{\rm ic}}$$

to be

$$P_{\rm ic,max} = \frac{375}{64\pi}\frac{1}{G^3\, M_{\rm ic}^2}\left(\frac{kT_{\rm ic}}{\mu_{\rm ic}\, m_{\rm H}}\right)^4 \propto \frac{1}{M_{\rm ic}^2}.$$

Isothermal Core Mass Limit (IV)

Now, estimate the mass, i.e. envelope pressure, that can be supported by the isothermal core: Starting again with

$$\frac{\mathrm{d}P}{\mathrm{d}M_r} = -\frac{GM_r}{4\pi r^4},$$

and assuming the pressure at the stellar surface to be zero, we have

$$\begin{aligned} P_{\mathrm{ic,env}} = \int_0^{P_{\mathrm{ic,env}}} \mathrm{d}P &= -\int_M^{M_{\mathrm{ic}}} \frac{GM_r}{4\pi r^4}\mathrm{d}M_r \\ &\simeq -\frac{G}{8\pi \langle r^4 \rangle}(M_{\mathrm{ic}}^2 - M^2), \end{aligned}$$

where M is the total mass of the star. Assuming $M_{\mathrm{ic}}^2 \ll M^2$, and making the crude approximation that $\langle r^4 \rangle \sim R^4/2$, we have

$$P_{\mathrm{ic,env}} \sim \frac{G}{4\pi}\frac{M^2}{R^4}.$$

Isothermal Core Mass Limit (V)

Making a rough estimate, $\rho_{\mathrm{ic,env}} \sim \frac{3M}{4\pi R^3}$, and using $P_{\mathrm{ic,env}} = \frac{\rho_{\mathrm{ic,env}}\, k\, T_{\mathrm{ic}}}{\mu_{\mathrm{env}}\, m_{\mathrm{H}}}$, we can estimate R with

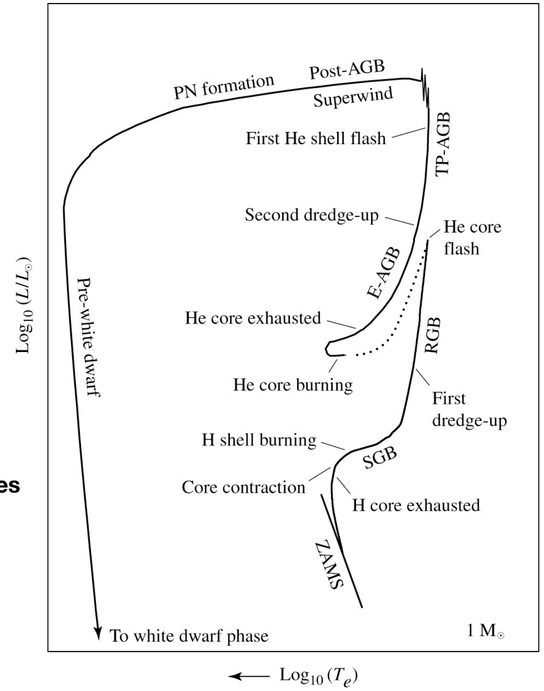
$$\begin{aligned} P_{\mathrm{ic,env}} &\sim \frac{G}{4\pi}\frac{M^2}{R^4} = \frac{GM}{3R}\rho_{\mathrm{ic,env}} = \frac{GM}{3R}\frac{P_{\mathrm{ic,env}}\,\mu_{\mathrm{env}}\, m_{\mathrm{H}}}{kT_{\mathrm{ic}}} \\ &\Rightarrow \quad R \sim \frac{1}{3}\frac{GM}{T_{\mathrm{ic}}}\frac{\mu_{\mathrm{env}}\, m_{\mathrm{H}}}{k} \\ \Rightarrow \quad P_{\mathrm{ic,env}} &\sim \frac{81}{4\pi}\frac{1}{G^3M^2}\left(\frac{kT_{\mathrm{ic}}}{\mu_{\mathrm{env}}\, m_{\mathrm{H}}}\right)^4. \end{aligned}$$

Finally, set the maximum pressure of the isothermal core equal to the pressure needed to support the overlying envelope and get

$$\frac{M_{\mathrm{ic}}}{M} \sim 0.54\left(\frac{\mu_{\mathrm{env}}}{\mu_{\mathrm{ic}}}\right)^2.$$

Evolution Off the Main Sequence

- **Evolutionary tracks and final stellar remanent depend on stellar masses**
  - ***M* < 8 *M*$_\odot$ turn into white dwarfs**
  - ***M* > 8 *M*$_\odot$ turn into supernovae**

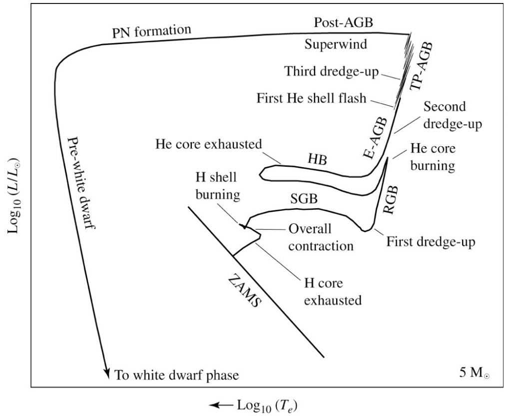


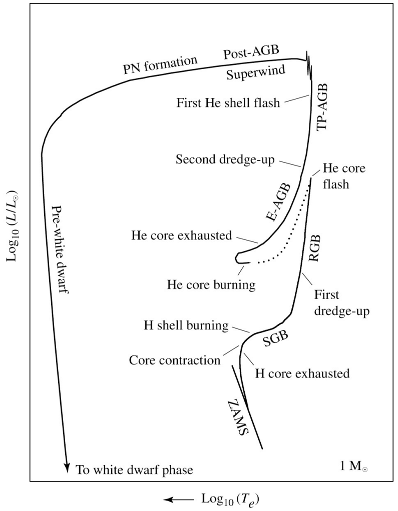
Suggested Guidelines

- **Evolution timescale**
  - Nuclear timescale ≫ Kelvin-Helmholtz timescale
- **Core activities**
  - Contraction: *T*↗ ~ *ε*↗ ~ *L*↗
  - Shell expansion: *T*↘ ~ *ε*↘ ~ *L*↘ (even ceases nuclear burning)
  - Cooling by neutrinos (temperature inversion)
  - Strongly electron-degenerate cores (*T* independent)
- **Envelope activities**
  - Contraction: *R*↘ ~ T_e↗ ~ bluer
  - Expansion: *R*↗ (L↗) ~ T_e↘ ~ redder
  - Cool surface ⇰ H$^-$ ions (large opacity) ⇰ onset of convection ⇰ mixing compositions

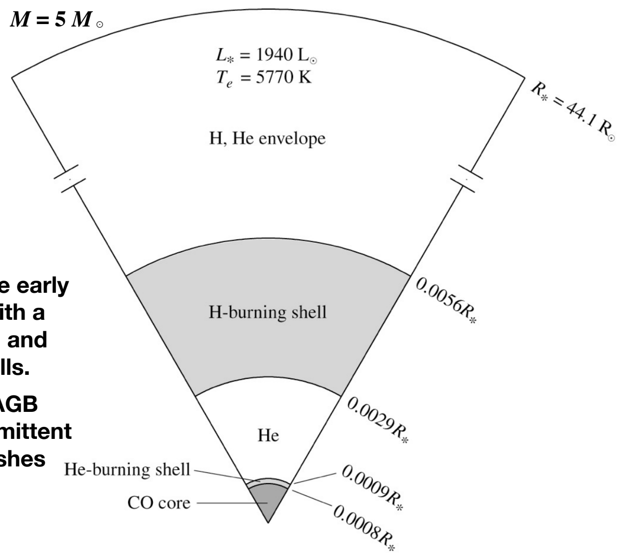
Comparison of 1 and 5 M☉

- **$M < 1.8\ M_\odot$ undergone He core flash**
  - Strongly electron-degenerate cores
  - Explosive energy release caused by the transition from e^- degeneracy pressure to thermal pressure via triple alpha process



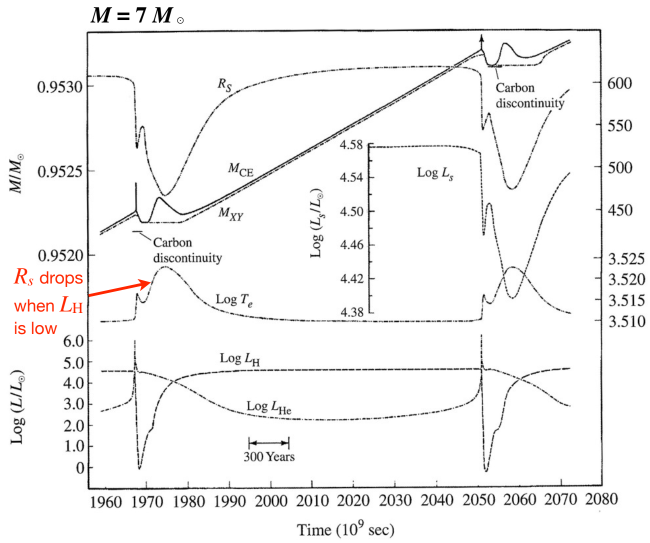


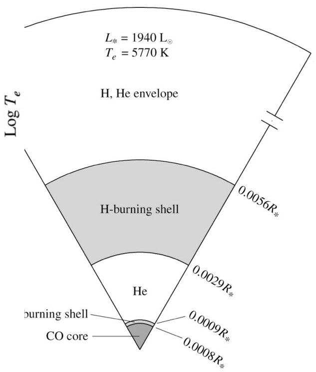
Asymptotic Giant Branch



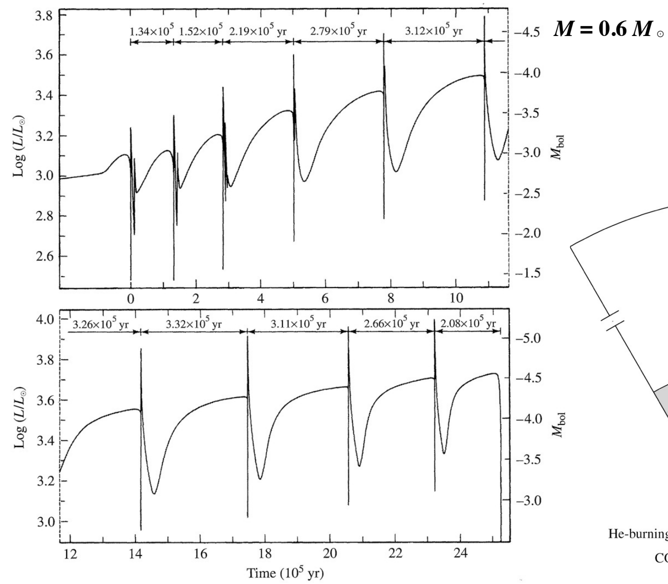
- **5 $M_\odot$ star on the early AGB (E-AGB) with a CO core and H- and He-burning shells.**
- **thermal-pulse AGB (TP-AGB): intermittent helium shell flashes**

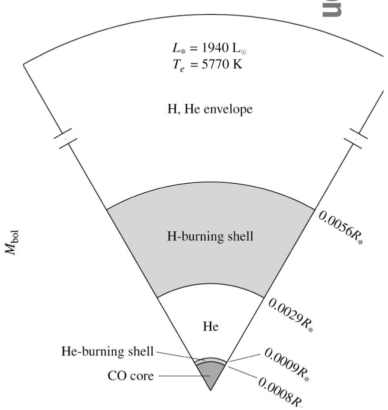
Mira Variables





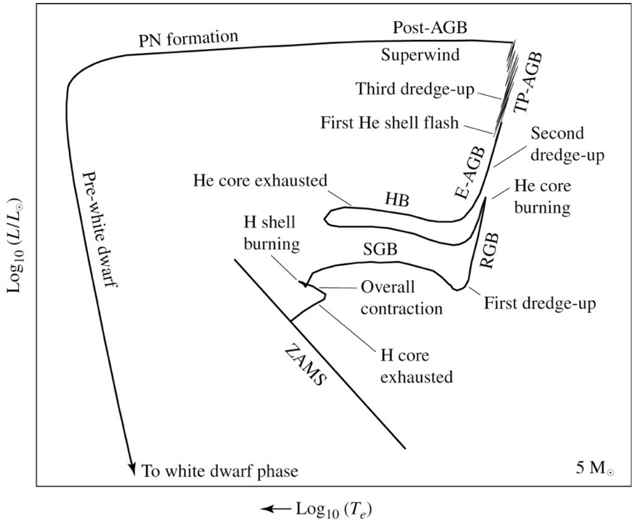
Mira Variables (TP-AGB)





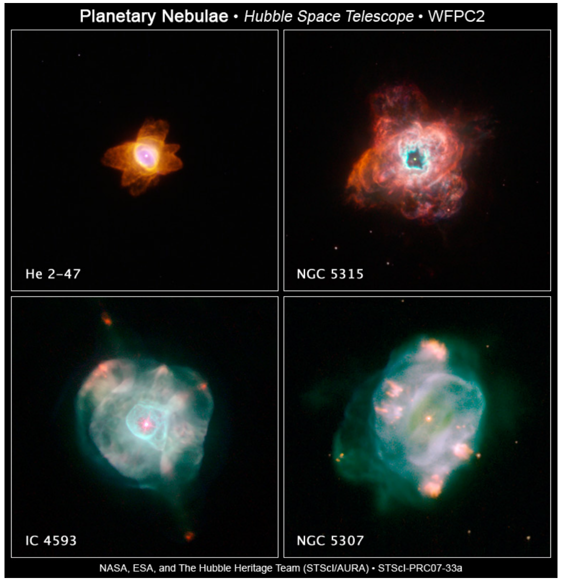
Third Dredge-Up & C Stars

- Third dredge-up
- C spectral type (carbon stars)
  - $X_C > X_O$ ⇨ SiC instead of SiO
- S spectral type
  - $X_C = X_O$



AGB Stars (Evolved Stars)

- **Asymptotic giant branch (AGB) stars**
  - Explosive phase in late stages of stars with $M < 8\ M_\odot$
  - Large mass-loss rate $\dot{M} \sim 10^{-4}\ M_\odot\ \mathrm{yr}^{-1}$
  - Enrich ISM with metals



Planetary Nebulae

- **Cat's Eye nebula (NGC 6543)**
  - bluish-green color owing to [O III] at 500.68 nm & 495.89 nm
  - [O II], [Ne III], etc

Helix nebula (NGC 7293)

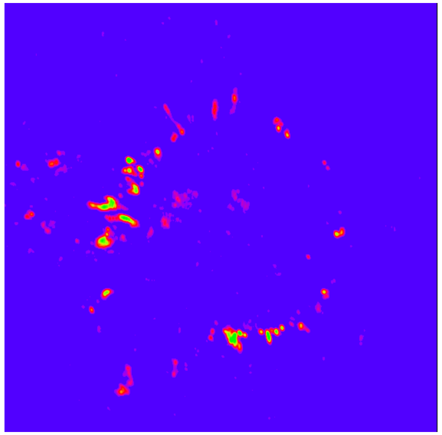
Planetary Nebulae

- **Egg Nebula 3**
  - Proto-planetary Nebula (PPN)

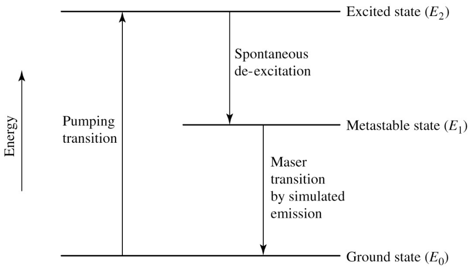
- **Menzel 3 (Mz 3)**
  - Outflow velocity ~ 1000 km s^{-1}

Masers in AGB Envelopes

43 GHz SiO maser emission around the Mira variable TX Cam

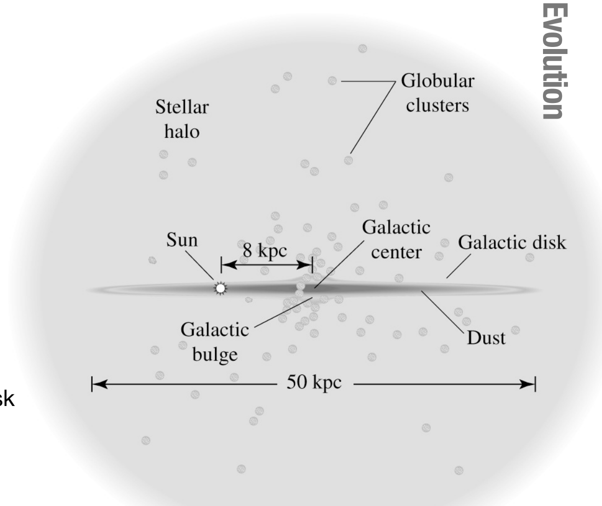


http://www.nrao.edu/pr/1999/txcam/



Stellar Populations

- **Originally identified with kinematically distinct groups of stars**
- **Population III**
  - First generation stars
  - Only H, He
- **Population II**
  - Metal poor
  - Found in the halo
- **Population I**
  - Our Sun
  - Metal rich
  - Found in the Galactic disk



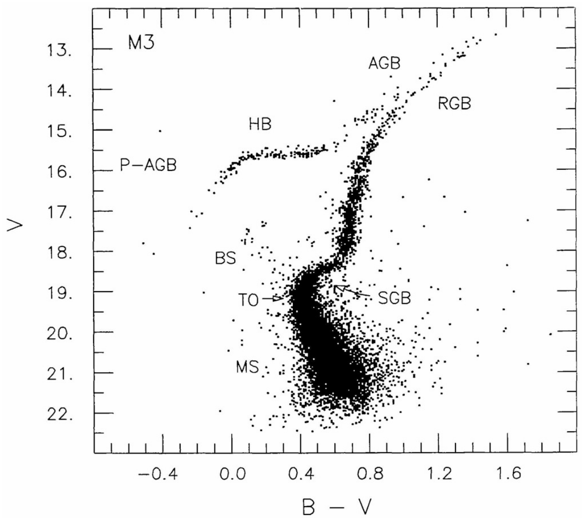
Stellar Clusters

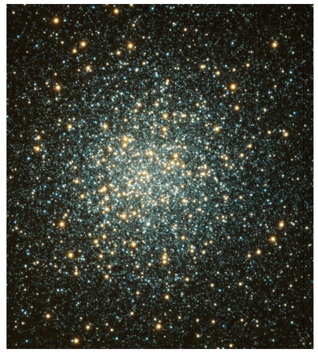
- **Open clusters**
  - Population I, young stars, gravitationally unbound
- **Globular clusters**
  - Population II, old stars, gravitationally bound, virialized

M13 (globular cluster)

Pleiades (open cluster)

Cluster Age & Turn-Off Point

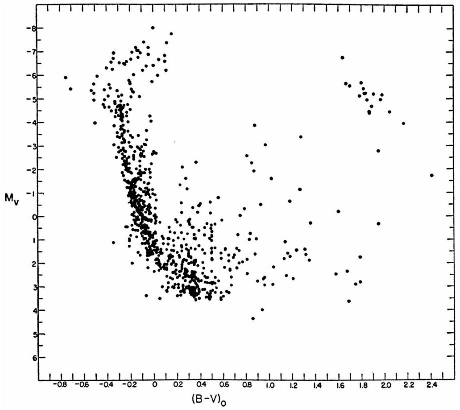




Hertzsprung Gap

h and Persei

Credit: Volker Wendel, Josef Popsel, Stefan Binnewies



- Caused by the short Kelvin-Helmholtz timescale following the hydrogen-depleted core
- Less pronounced for $M < 1.25\,M_\odot$

Comparison of Cluster Ages