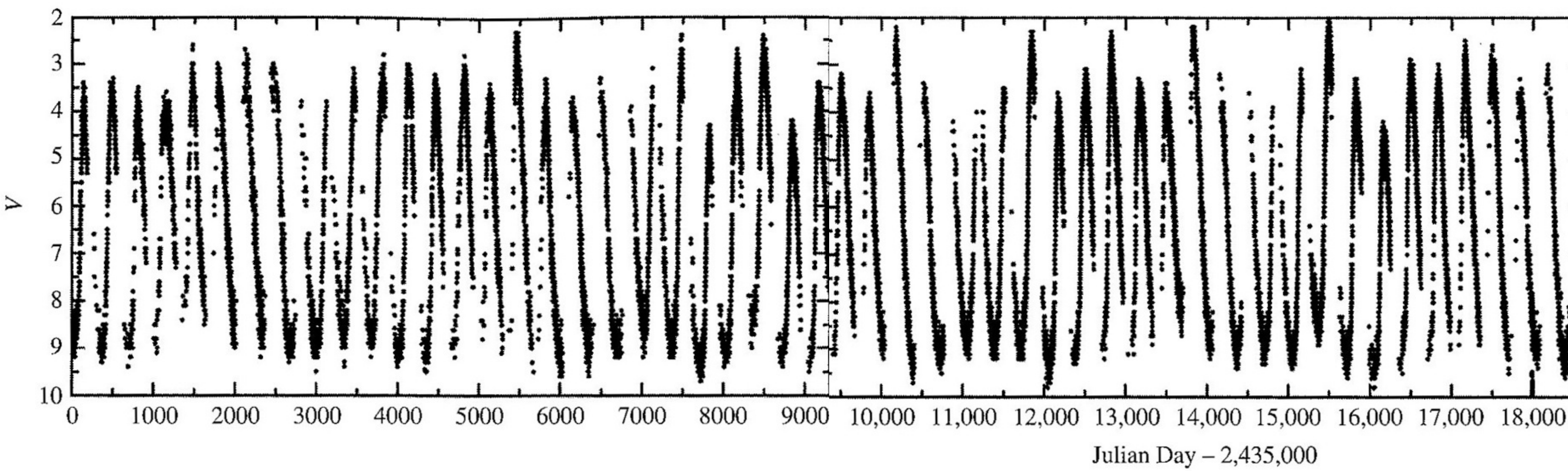
Chapter 14
Stellar Pulsation

Observations of pulsating stars
The physics of stellar pulsation
Modeling stellar pulsation
Nonradial stellar pulsation
Helioseismology and Asteroseismology

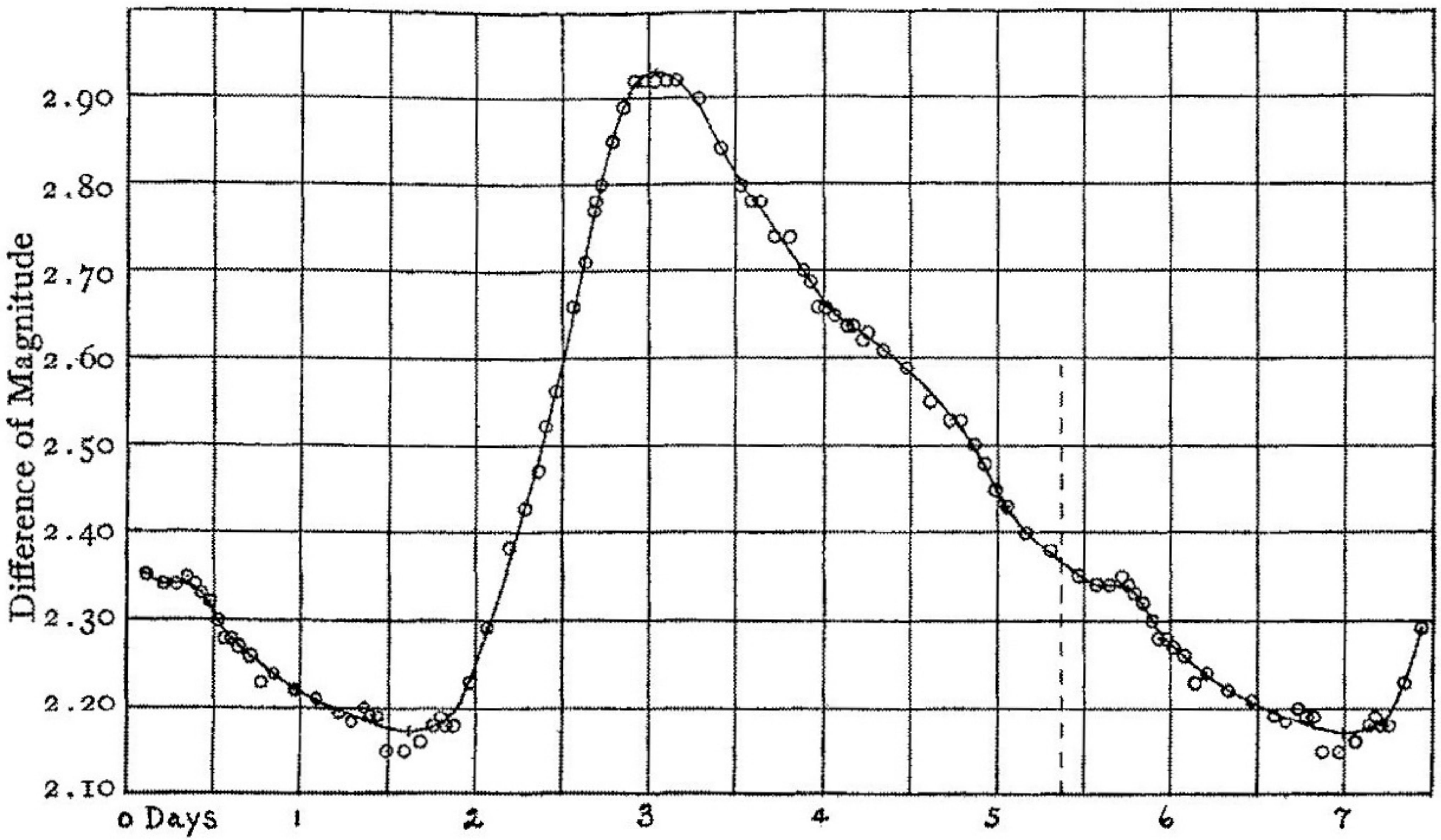
Light Curve of Mira

- Mira, prototype of the long-period variables



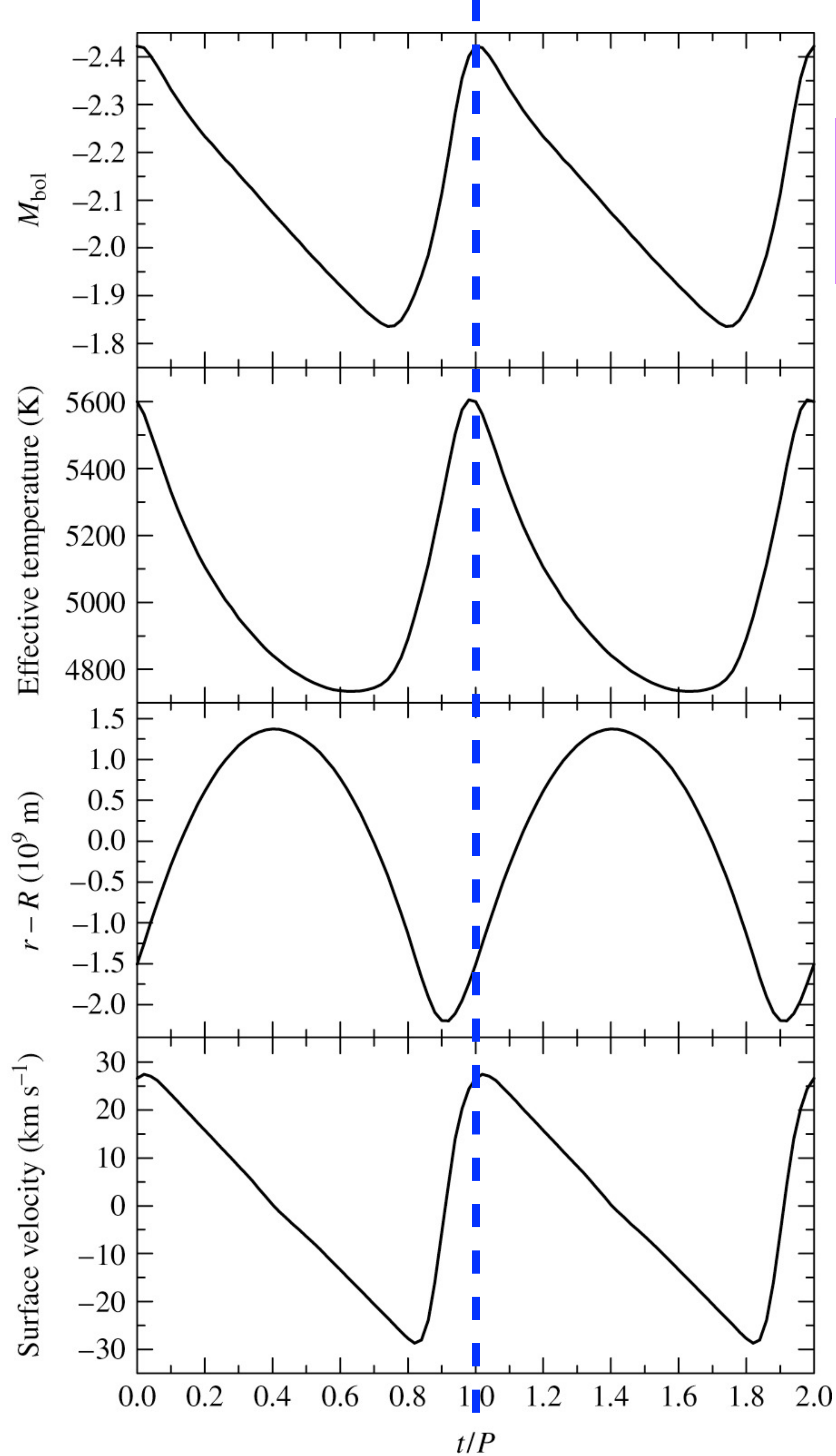
Light Curve of δ Cephei

- **A classical Cepheid**



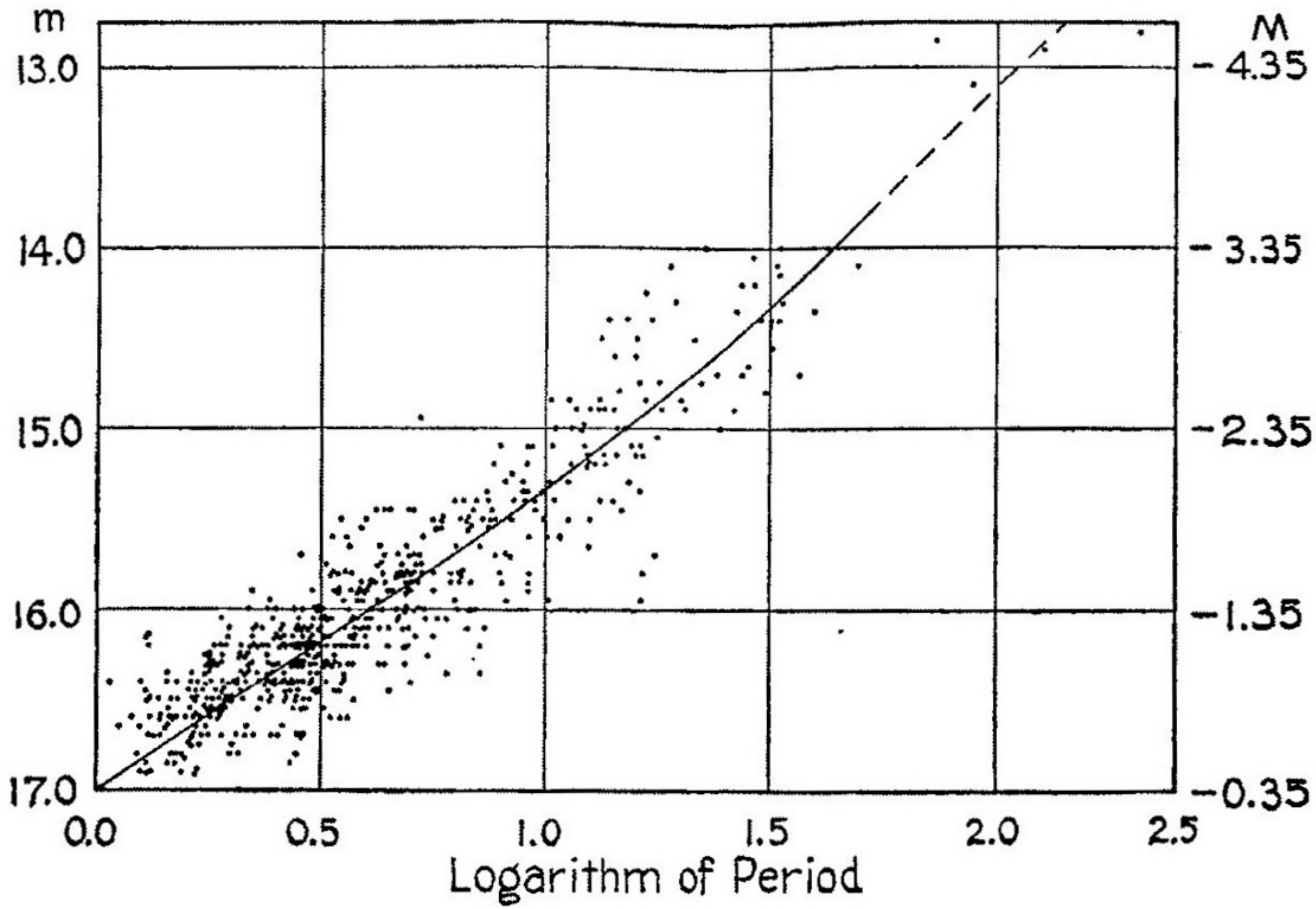
Pulsation Properties of δ Cephei

- Large T variation leads to large variation in luminosity
- Phase lag reflects underlying pulsating mechanism: maximum luminosity occurs behind minimum radius

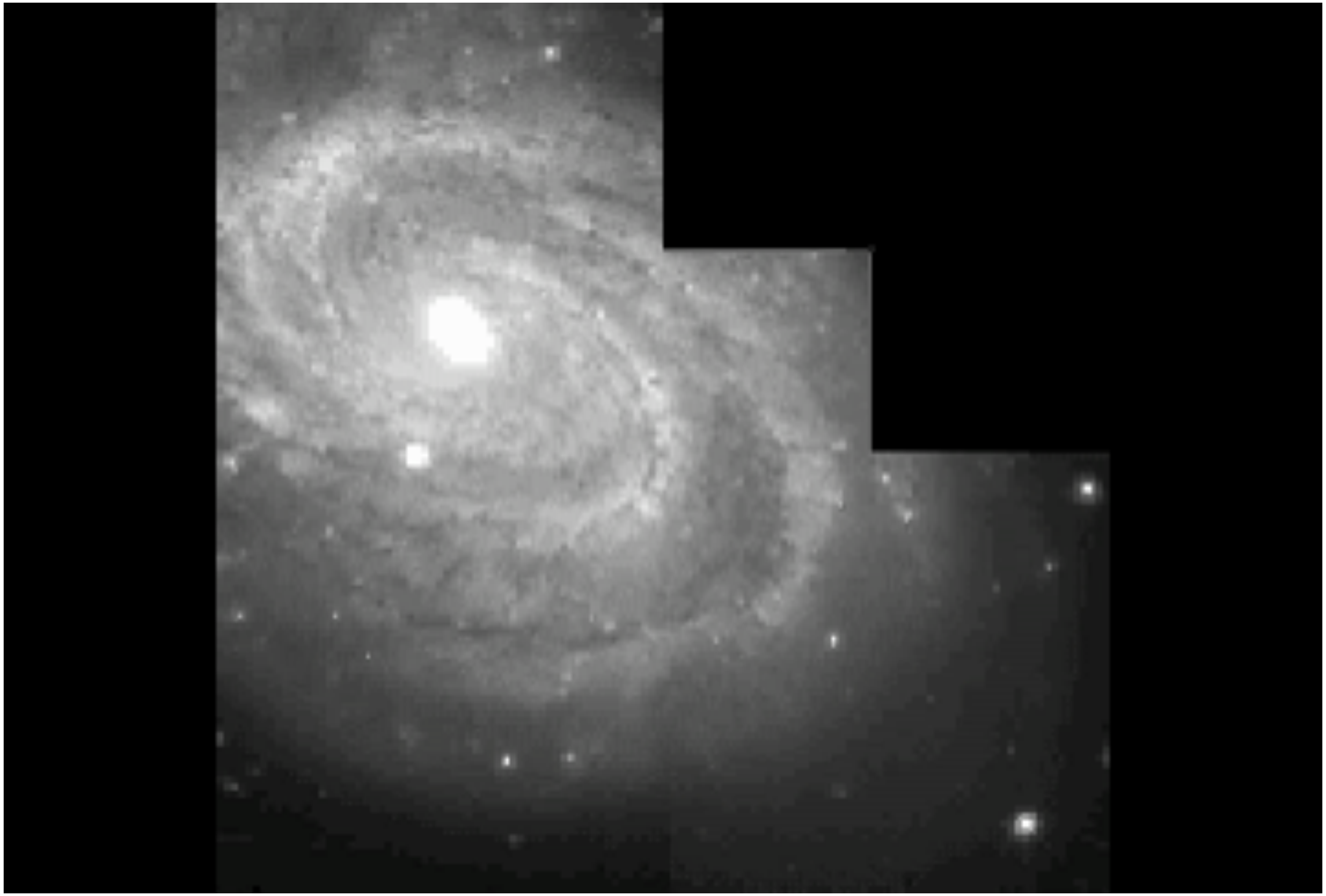


Classical Cepheids in SMC

- **Period-luminosity relation for classical Cepheids in the Small Magellanic Cloud**



Cepheids in NGC 4603



Credit: Jeffrey Newman (UC Berkeley) & NASA

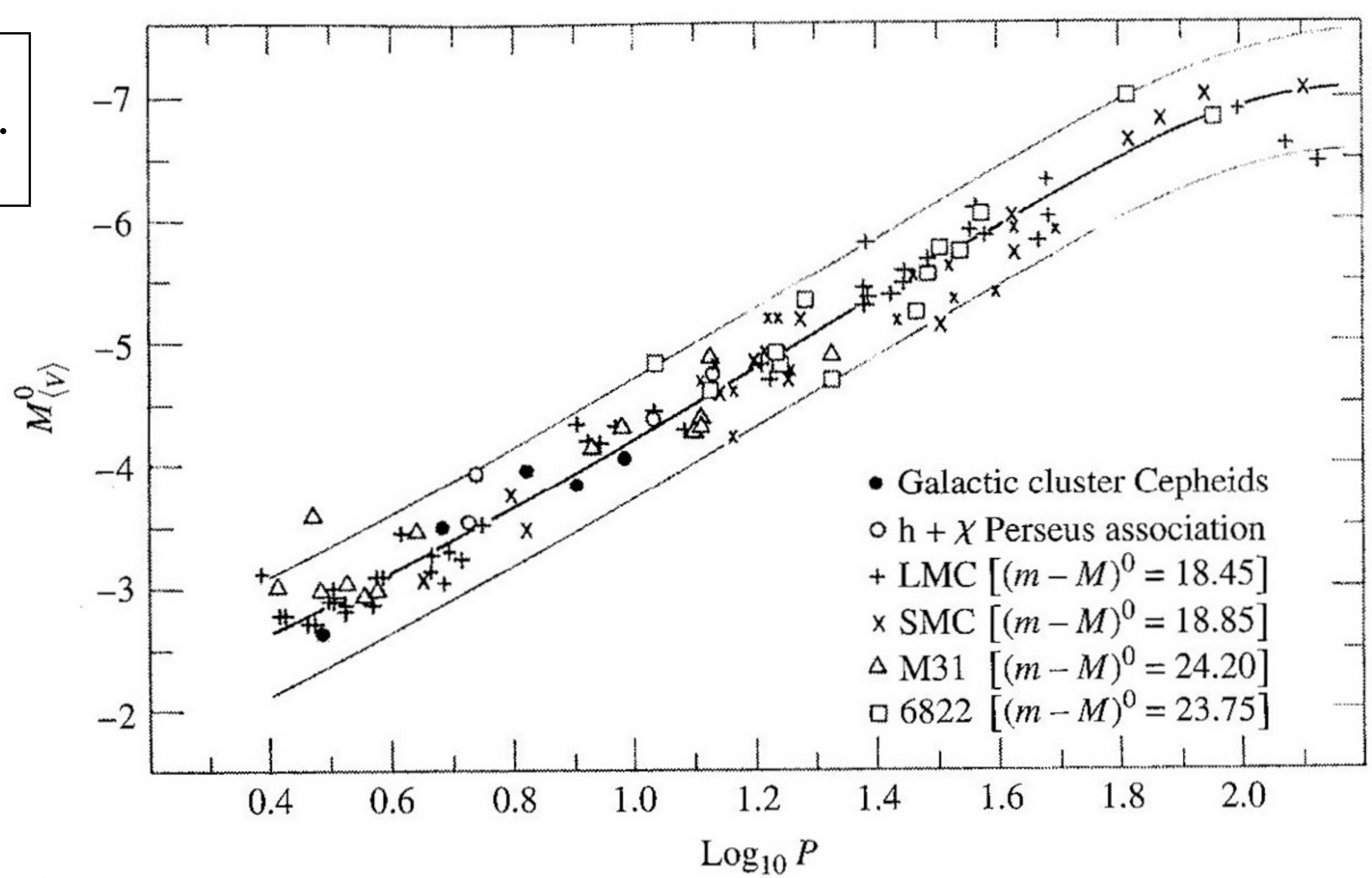
Classical Cepheids in SMC

Period-luminosity relation of classical Cepheids for the V band

$$M_{\langle V\rangle} = -2.81 \log P_{\rm d} - 1.43$$

In terms of the average luminosity, the relation is given by

$$\log \frac{\langle L\rangle}{L_\odot} = 1.15 \log P_{\rm d} + 2.47.$$



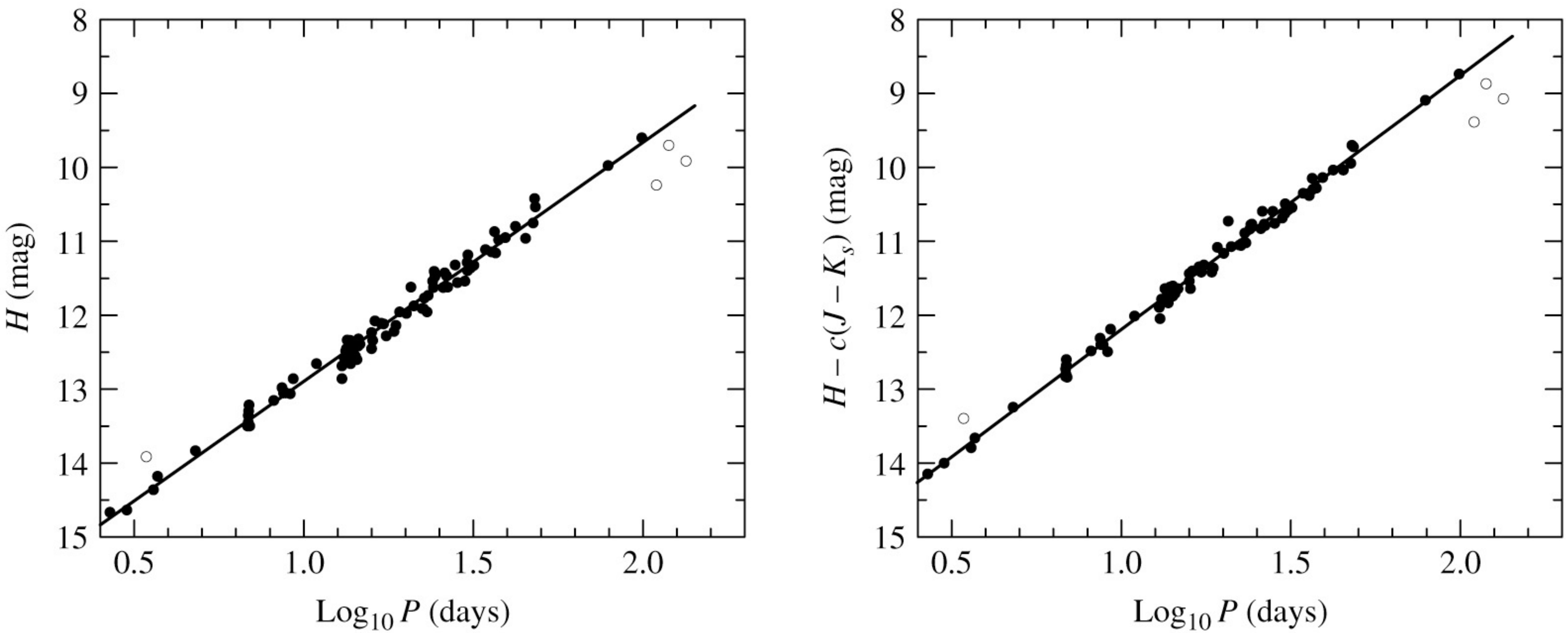
Infrared Cepheid P-L relation

Reduce interstellar extinction by observing in the infrared H band ($\lambda = 1.654\ \mu$m)

$$H = -3.234 \log P_\mathrm{d} + 16.079,$$

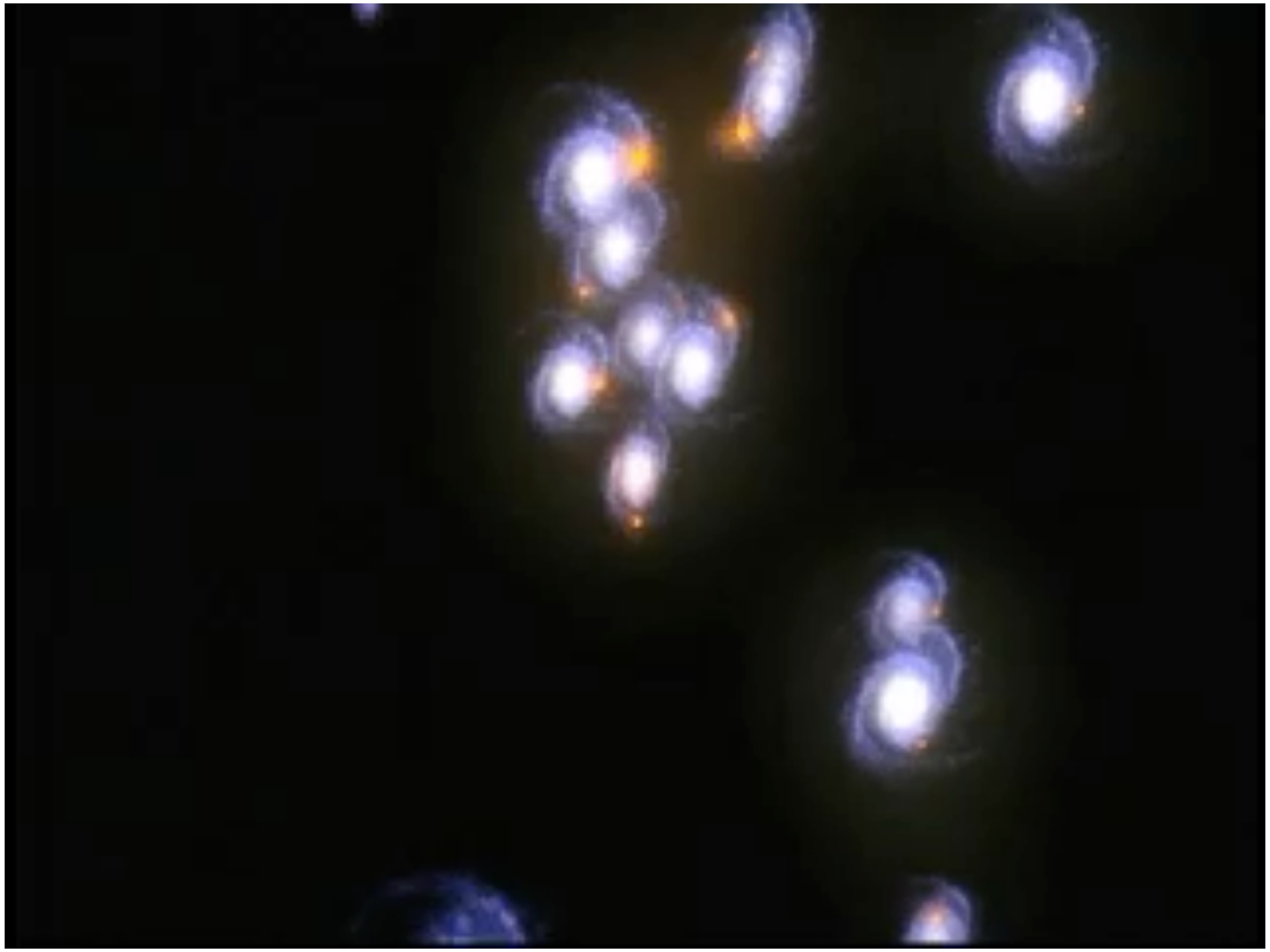
and further improve with infrared color index $J - K_s$

$$H = -3.428 \log P_\mathrm{d} + 1.54\langle J - K_s \rangle + 15.637.$$

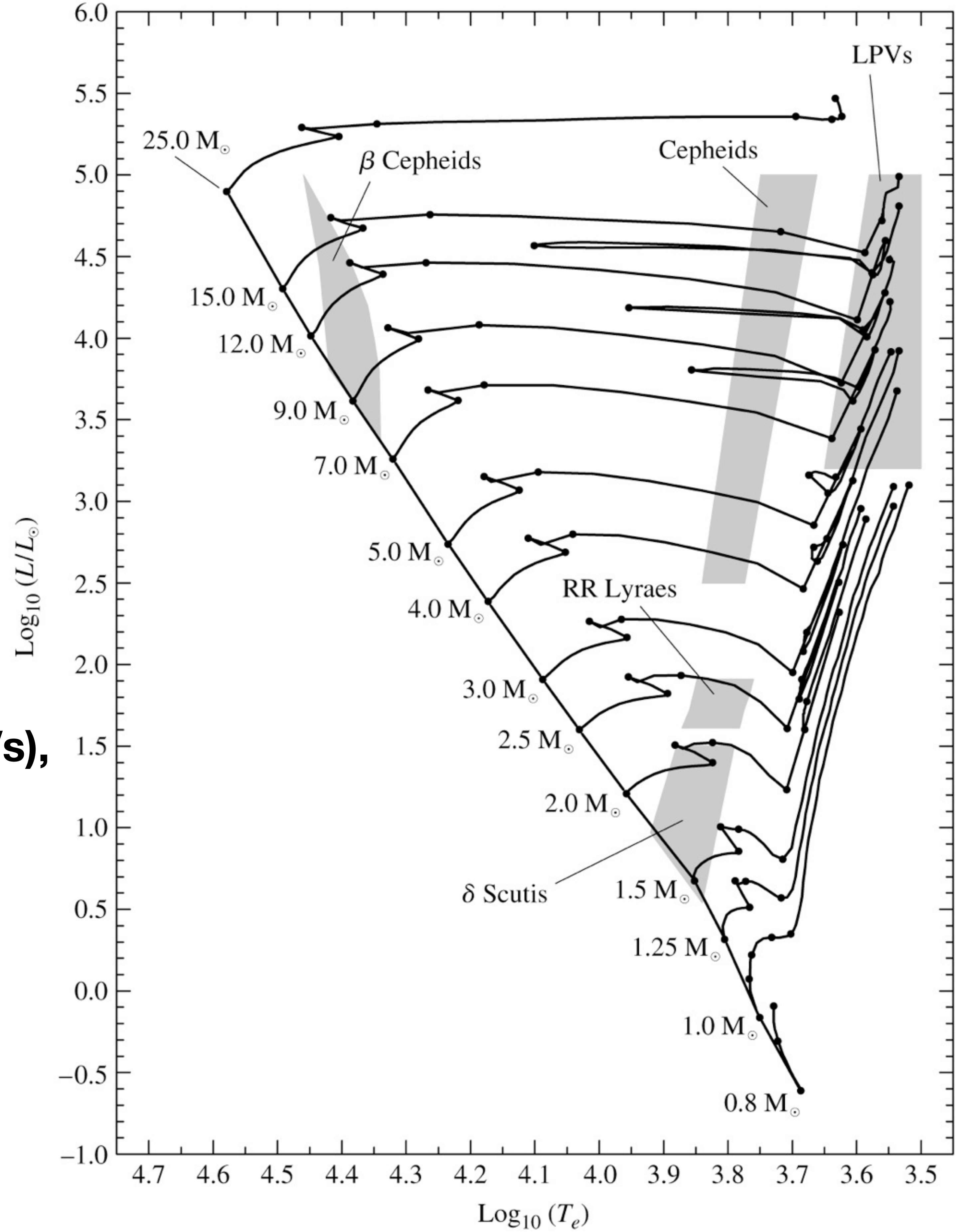


Standard Candles

- **Apply period-luminosity relation and $F = L/4\pi d^2$, we can get the distances of galaxies**



Pulsating Stars



- **Long period variables (LPVs), e.g. Mira**
- **Classical Cepheids**
- **W Virginis**
- **RR Lyrae**
- **δ Scuti**

Pulsating Stars

Type	Range of Periods	Population Type	Radial/Nonradial
Long-Period Variables	100-700 days	I, II	R
Classical Cepheids	1-50 days	I, II	R
W Virginis	2-45 days	II	R
RR Lyraes	1.5-24 hours	II	R
δ Scuti	1-3 hours	I	R, NR
β Cephei	3-7 hours	I	R, NR
ZZ Ceti stars	100-1000 sec	I	NR

Period-Density Relation (I)

The radial oscillation of a pulsating star are the result of sound waves resonating in the star's interior. The pulsation period, Π, can be approximated by the time it would take for a sound wave to cross the diameter of a toy model star. Using the hydrostatic equilibrium

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM_r\rho}{r^2} = -\frac{G(\frac{4}{3}\pi r^3\rho)\rho}{r^2} = -\frac{4}{3}\pi G\rho^2 r,$$

and integrating with the boundary condition that $P = 0$ at the surface $r = R$, we have

$$P(r) = \frac{2}{3}\pi G\rho^2(R^2 - r^2).$$

The pulsation period is roughly

$$\Pi \approx 2\int_0^R \frac{\mathrm{d}r}{v_s} \approx 2\int_0^R \frac{\mathrm{d}r}{\sqrt{\frac{2}{3}\gamma\pi G\rho(R^2 - r^2)}},$$

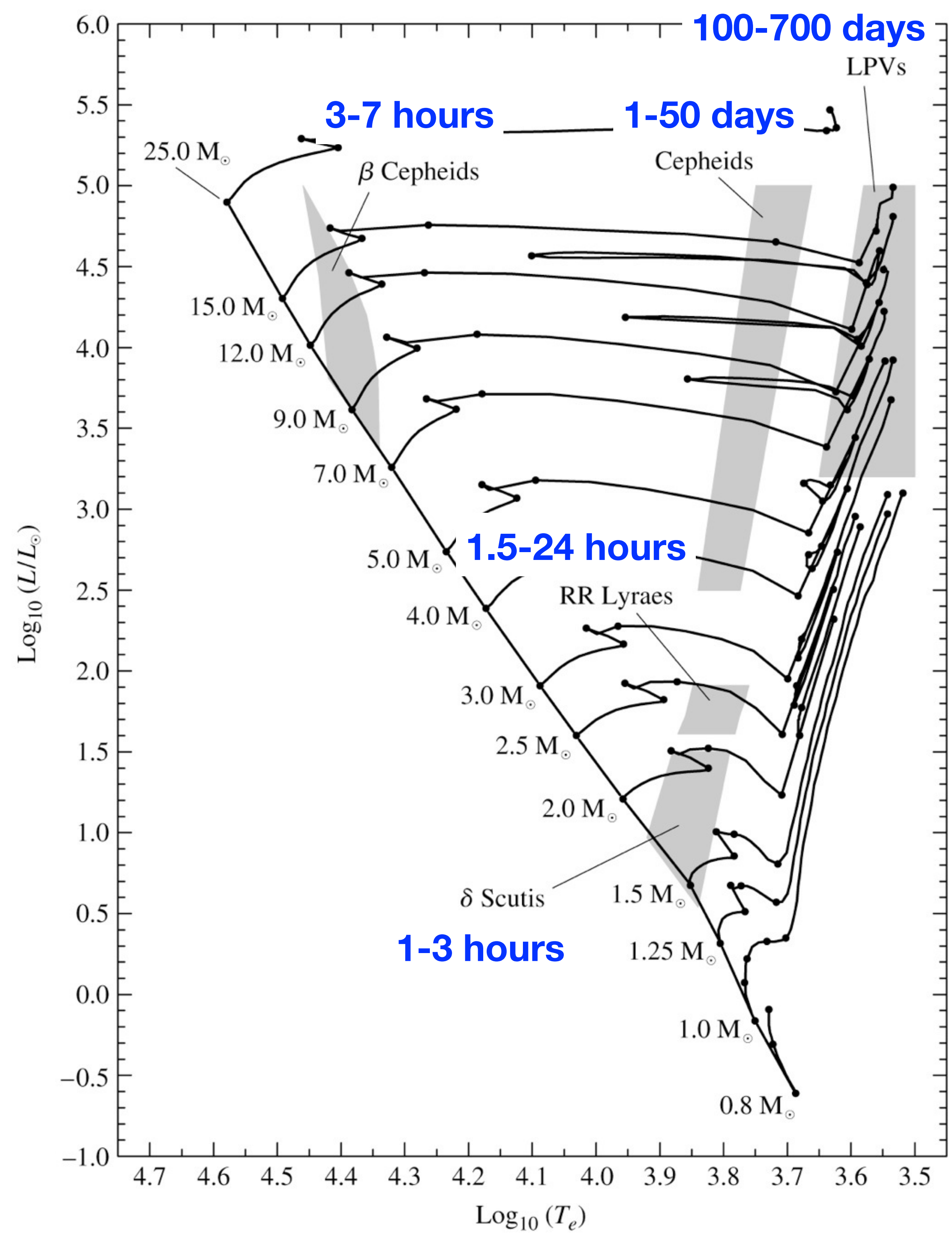
Period-Density Relation (II)

Since

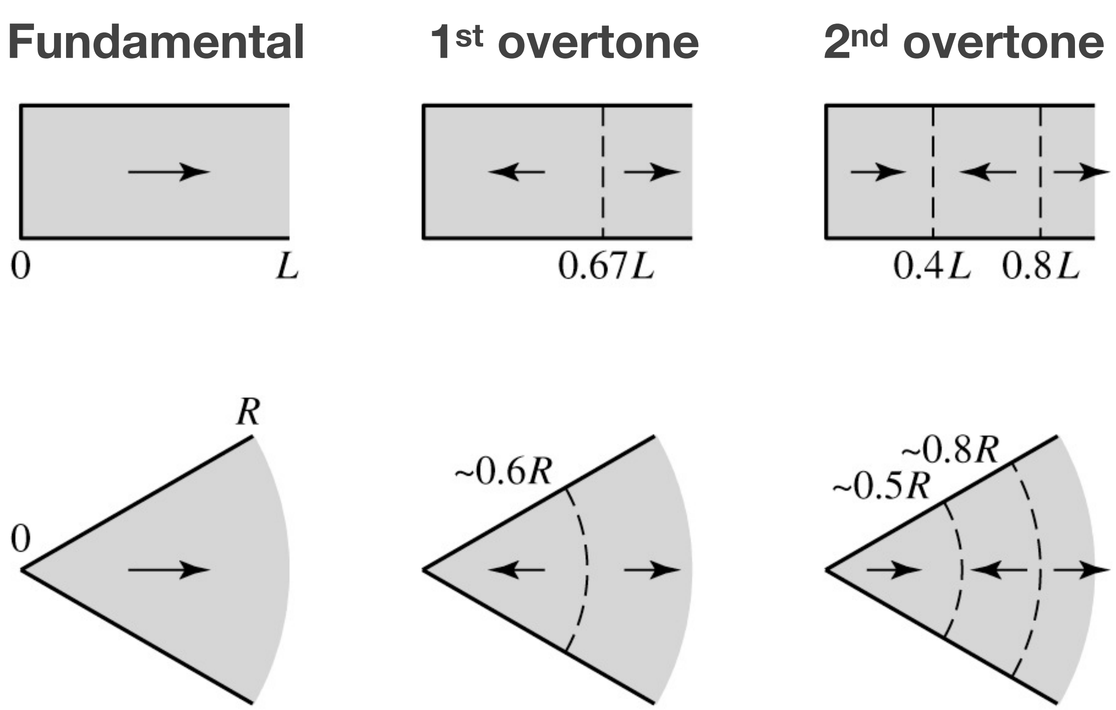
$$\int_0^R (R^2 - r^2)^{-1/2} \mathrm{d}r = \frac{\pi}{2},$$

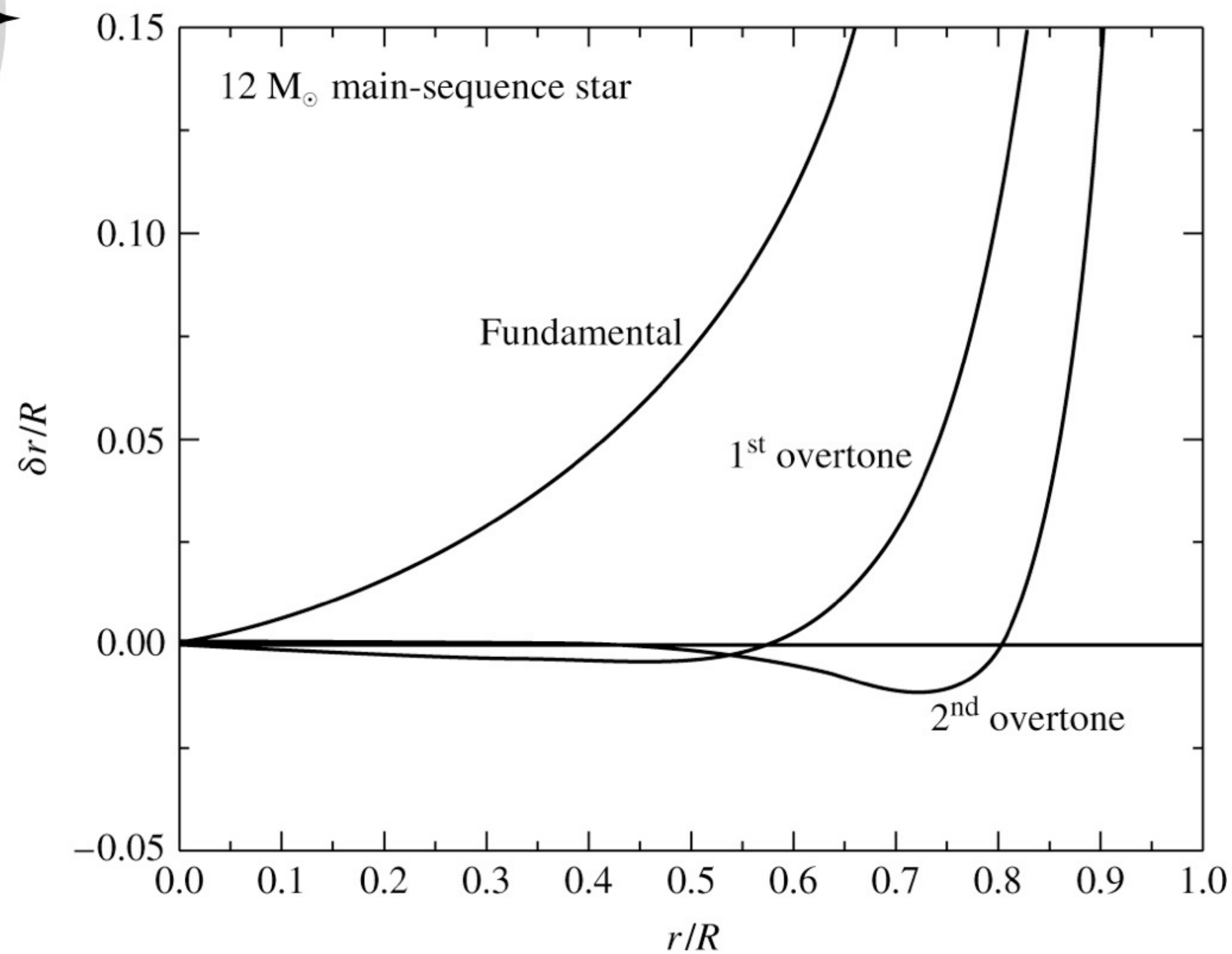
we arrive at the **period-mean density relation**

$$\boxed{\Pi \approx \sqrt{\frac{3\pi}{2\gamma G\rho}}} \propto \rho^{-1/2}$$



Radial Modes of Pulsation

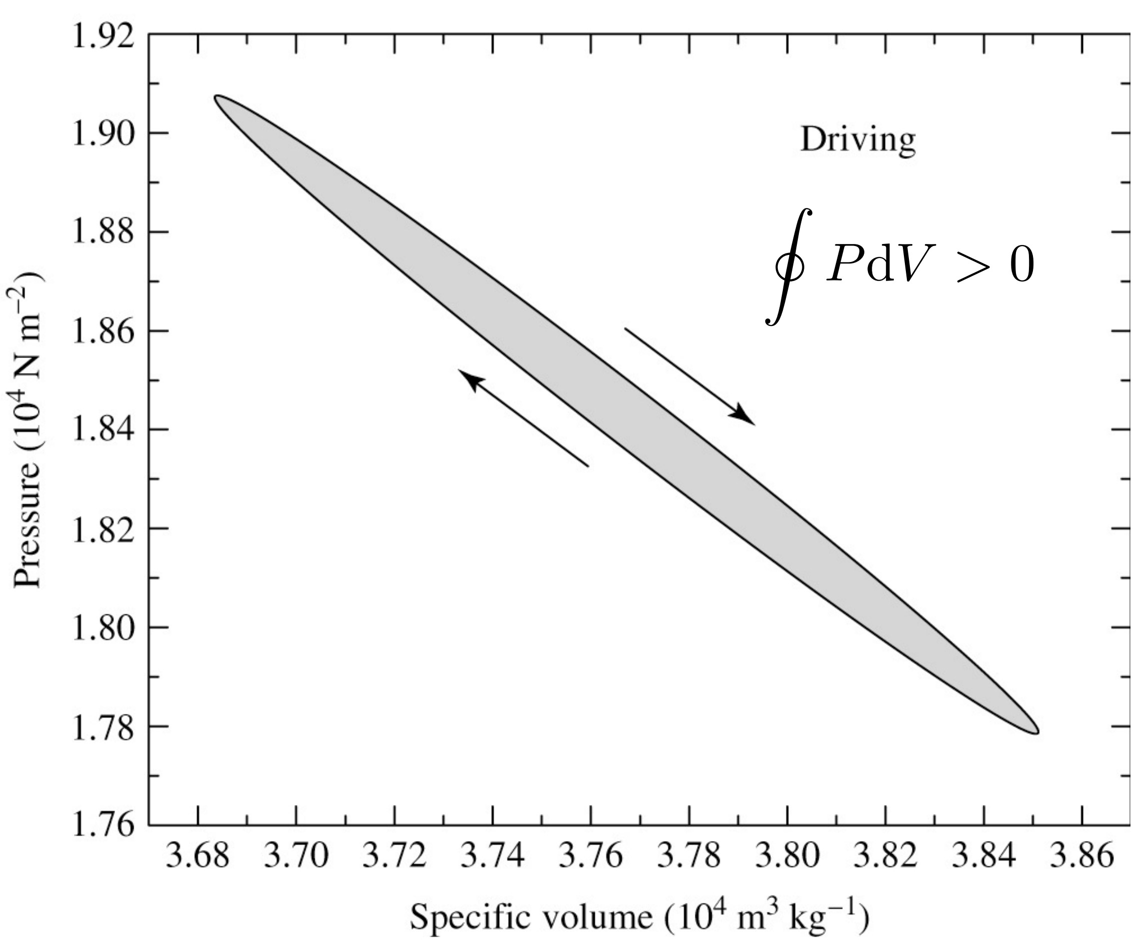


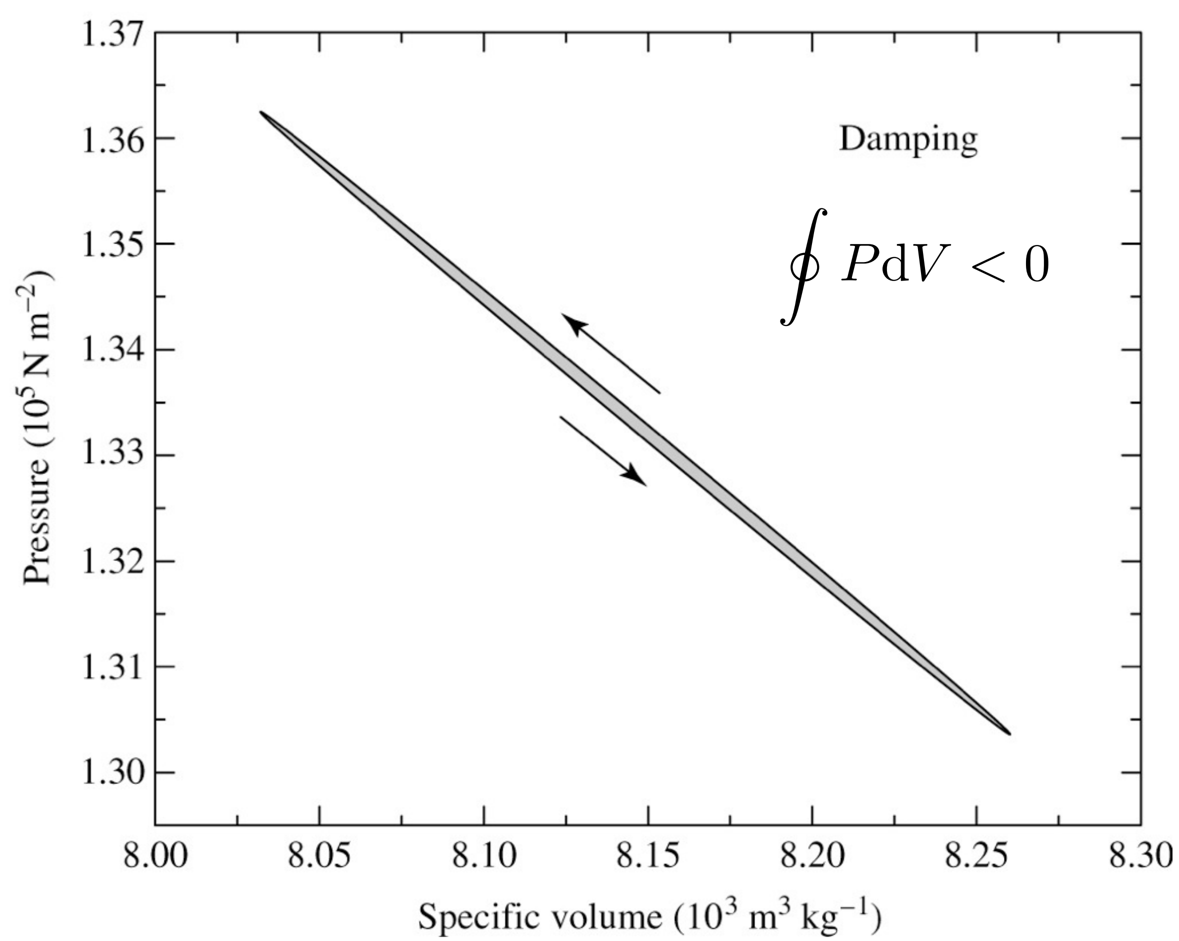


- **Radial standing wave with one open end, similar to an organ pipe**

What Drives Stellar Pulsation

- **Eddington's thermodynamic heat engine**
  - **Nuclear mechanism: ε mechanism**
    - Insignificant to drive pulsation: displacement, $\delta r/R$, has a node at the center
  - **Piston layer in stellar interior: κ and γ mechanisms**
    - Opacity must increase with compression (partial ionization zones)
    - Kramers law: $\kappa \propto \rho/T^{3.5}$

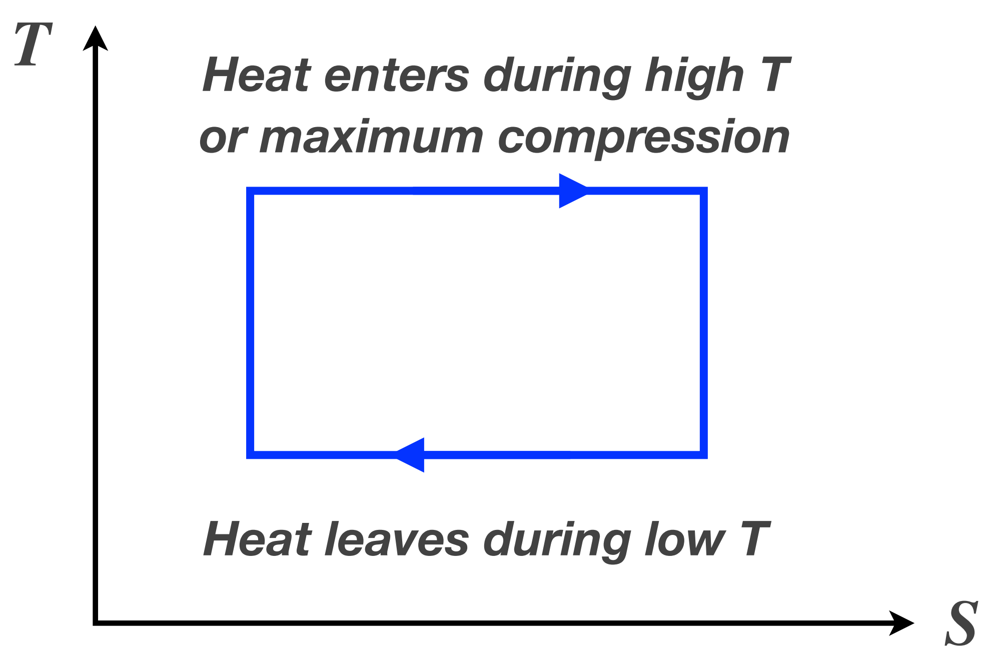


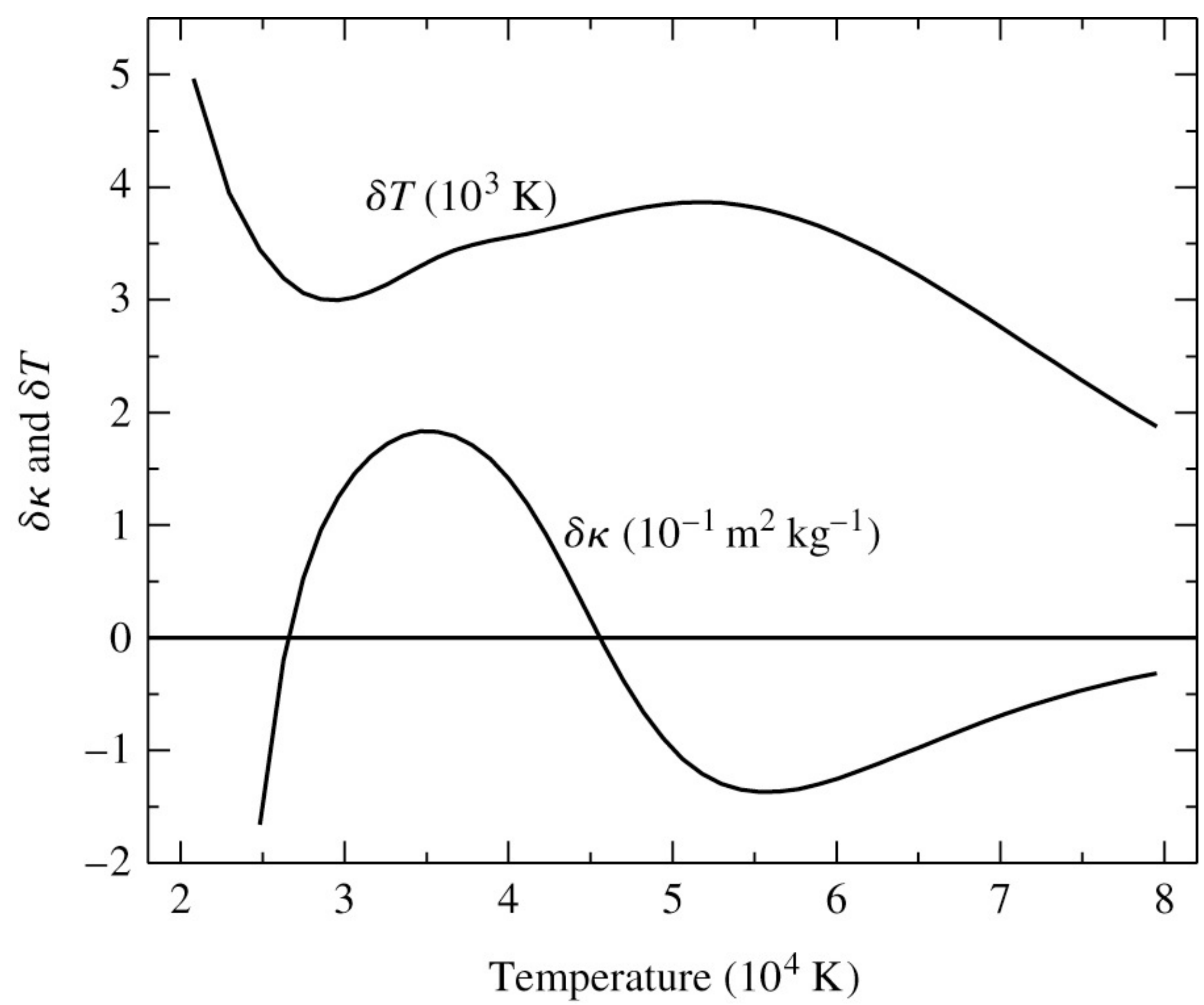


κ and γ Mechanisms

- **Opacity Effects: κ and γ mechanisms**
  - Partial ionization zones: opacity increases with compression (κ mechanism)
  - Small T increment due to large heat capacities ⇨ $\kappa \propto \rho/T^{3.5}$ ↗
  - Lower T ⇨ heat exchanges with adjacent stellar layers (γ mechanism)

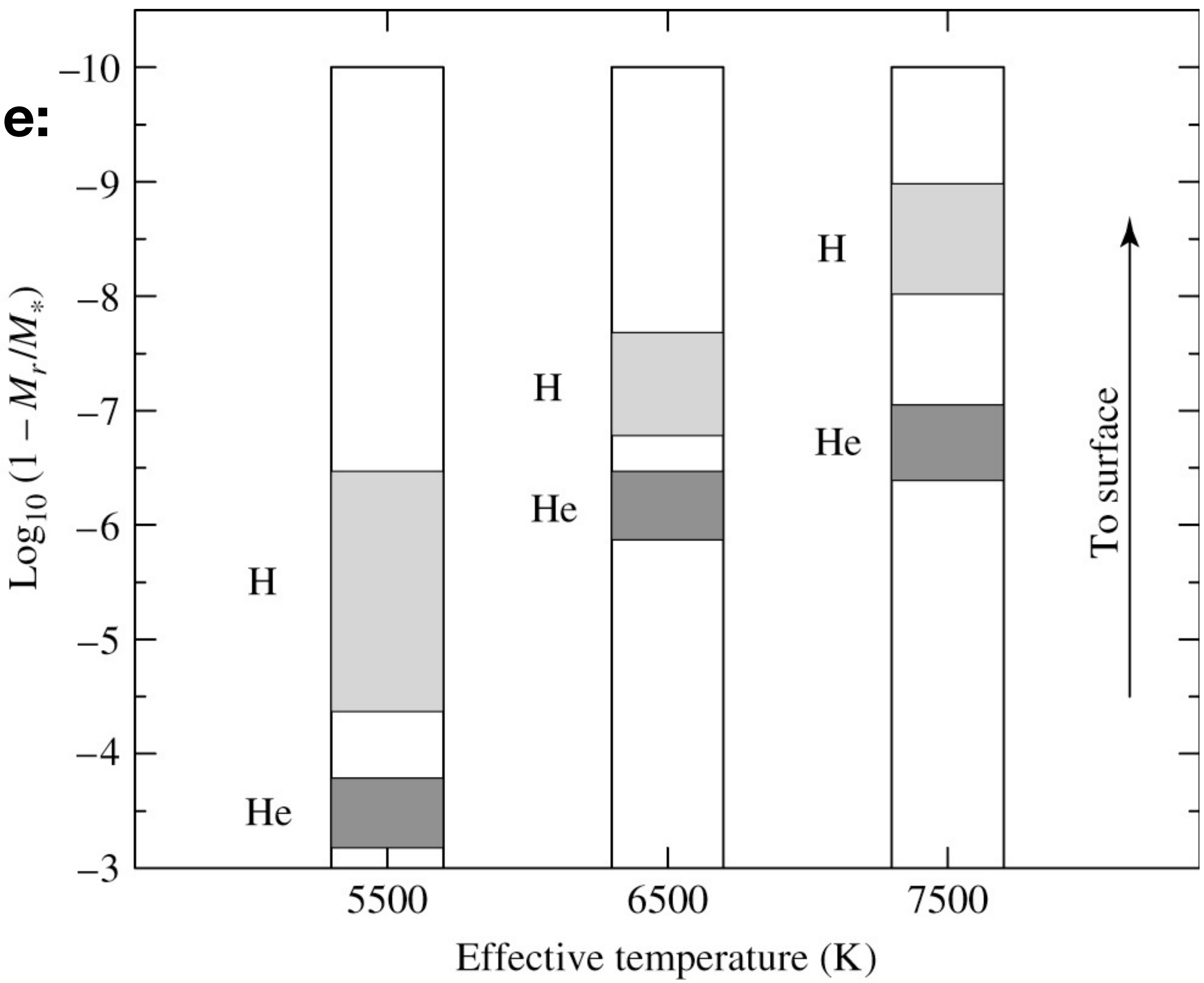
Carnot cycle in heat absorbing





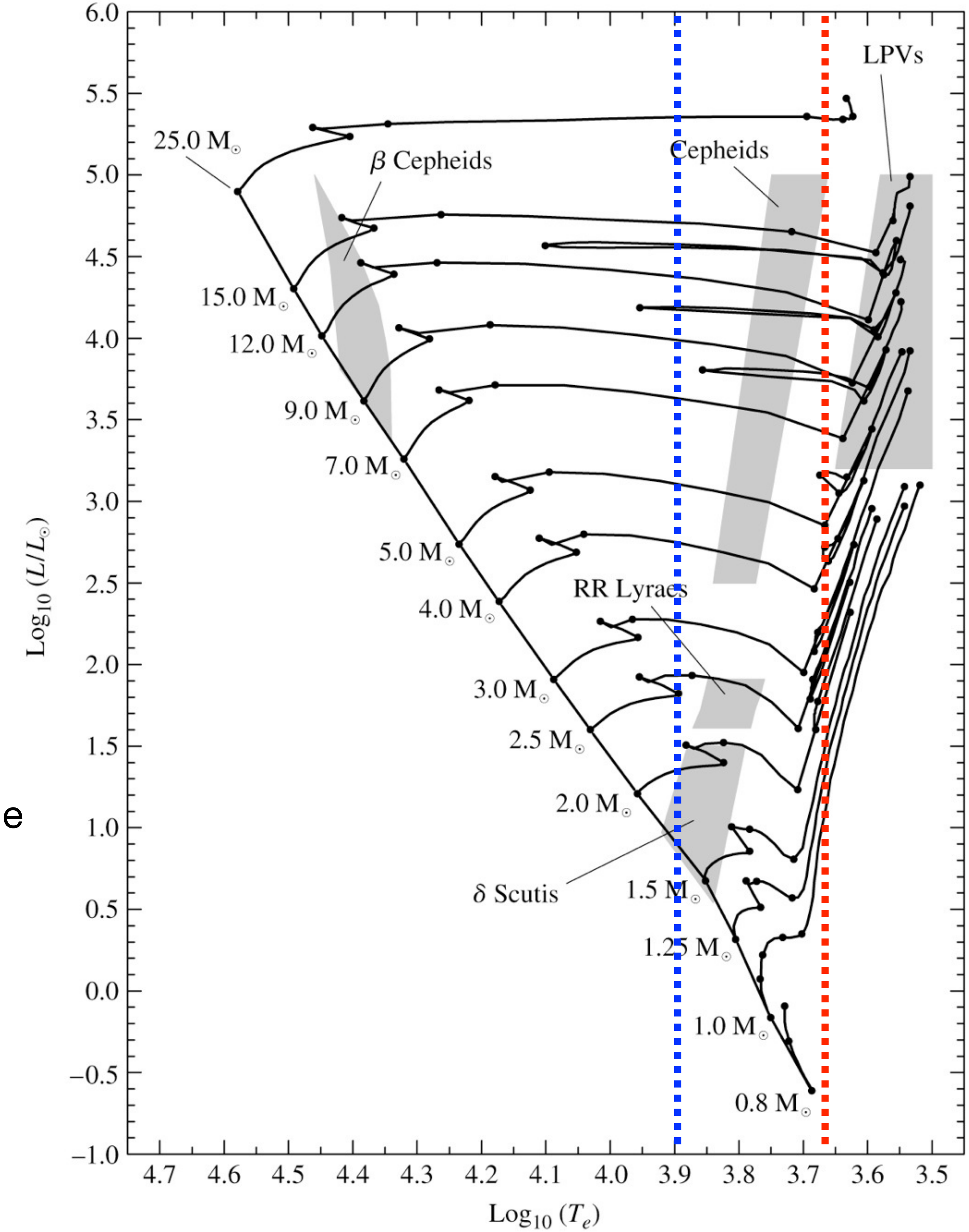
κ Mechanism

- **H partial ionization zone:**
  - **H I → H II & He I → He II**
  - $(1\text{-}1.5)\times10^4$ K
  - **Resulting phase lag**
- **He partial ionization zone:**
  - **He II → He III**
  - 4×10^4 K
  - **Primary for pulsations**



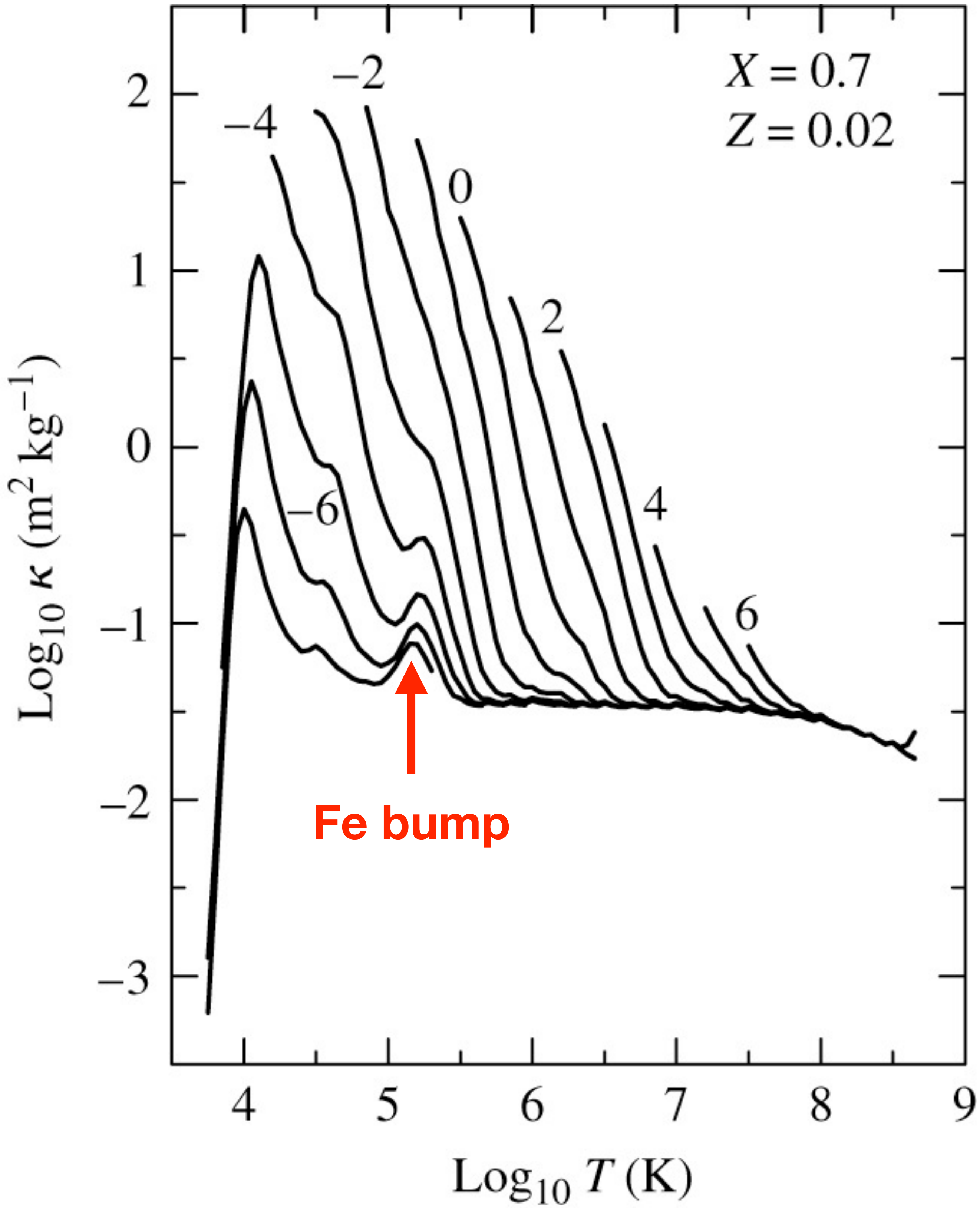
Instability Strip

- **Blue edge: ~7500 K**
  - Shallow He II ionization zone
- **Red edge:**
  - Cool surface resulting in convection damping



β Cephei Stars

- Early B stars
  - T_e ~ (2-3)×10^4 K
  - MK class: III, IV, V
- Pulsation caused by iron bump in opacity around T~10^5 K



Modeling Stellar Pulsation (I)

Equation of motion can be written as

$$\rho \frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = -G \frac{M_r \rho}{r^2} - \frac{\mathrm{d}P}{\mathrm{d}r},$$

which is *nonlinear* and has to be solved numerically.

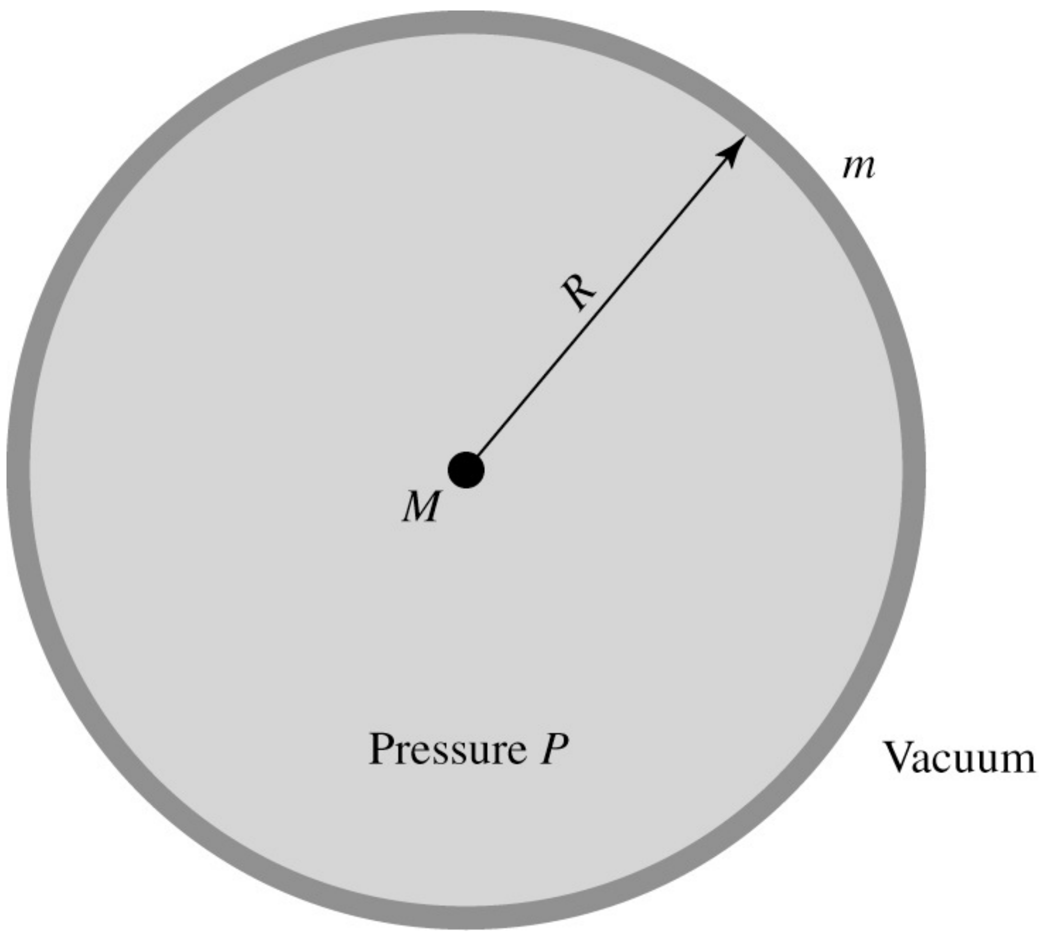
An alternative is to linearize the differential equation.

Consider a one-zone model of a toy pulsating star, consisting of a central point mass, M, and a single think spherical shell of mass m and radius R. We have

$$m \frac{\mathrm{d}^2 R}{\mathrm{d}t^2} = -\frac{GMm}{R^2} + 4\pi R^2 P.$$

At the equilibrium state, we shall have

$$\frac{GMm}{R_0^2} = 4\pi R_0^2 P_0.$$



Modeling Stellar Pulsation (II)

Now, let $R = R_0 + \delta R$ and $P = P_0 + \delta P$. Using the first-order approximation

$$\frac{1}{(R_0 + \delta R)^2} \approx \frac{1}{R_0^2}\left(1 - 2\frac{\delta R}{R_0}\right),$$

we obtain

$$m\frac{\mathrm{d}^2(\delta R)}{\mathrm{d}t^2} = -\frac{GMm}{R_0^2} + \frac{2GMm}{R_0^3}\delta R + 4\pi R_0^2 P_0 + 8\pi R_0 P_0 \delta R + 4\pi R_0^2 \delta P,$$

where $\mathrm{d}^2 R_0/\mathrm{d}t^2 = 0$ at the equilibrium state. Eliminating equal terms given by the equilibrium state, we have

$$m\frac{\mathrm{d}^2(\delta R)}{\mathrm{d}t^2} = \frac{2GMm}{R_0^3}\delta R + 8\pi R_0 P_0 \delta R + 4\pi R_0^2 \delta P.$$

Assume the oscillations are *adiabatic*, that is, $PV^\gamma = \text{constant} \propto PR^{3\gamma}$. The linearized version of this expression is

$$\frac{\delta P}{P_0} = -3\gamma\frac{\delta R}{R_0}.$$

Modeling Stellar Pulsation (III)

Substituting δP with δR and rearranging the equation of motion, we have

$$\frac{\mathrm{d}^2(\delta R)}{\mathrm{d}t^2} = -(3\gamma - 4)\frac{GM}{R_0^3}\delta R.$$

If $\gamma > 4/3$, the above equation is for simple harmonic motion with the solution $\delta R = A\sin(\omega t)$, where A is the amplitude and ω is the pulsation frequency.

$$\omega^2 = (3\gamma - 4)\frac{GM}{R_0^3}.$$

Finally, the pulsation period of the one-zone model is just

$$\Pi = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{4}{3}\pi G\rho_0(3\gamma - 4)}},$$

where $\rho_0 = M/\frac{4}{3}\pi R_0^3$. Recall that for an ideal monoatomic gas, $\gamma = 5/3$.

Dynamical Stability

Recall that

$$\frac{\mathrm{d}^2(\delta R)}{\mathrm{d}t^2} = -(3\gamma - 4)\frac{GM}{R_0^3}\delta R.$$

If $\gamma < 4/3$, the solution is now $\delta R = Ae^{-\kappa t}$, the star *collapses* and no pulsation will be present.

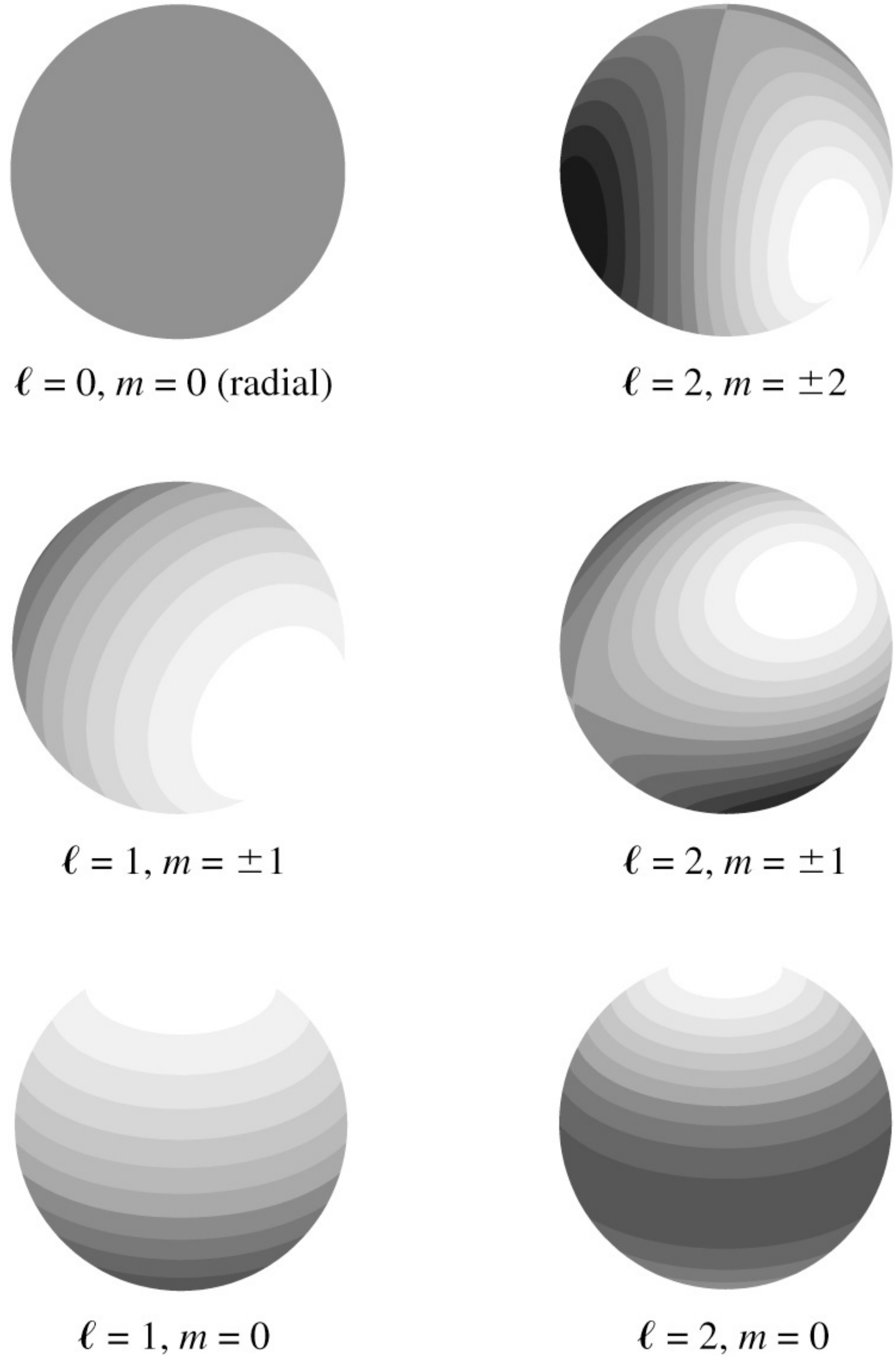
In reality, Eddington's valve mechanism involves heat exchanges, that is, the piston layer is not adiabatic. In the case of *nonadiabatic* oscillations, the time dependence of the pulsation is taken to be $\delta R \propto e^{i\sigma t}$, where $\sigma = \omega + i\kappa$. In this expression, ω is the usual pulsation frequency, while κ is a stability coefficient that characterizes the time for the growth or decay of the oscillation.

Nonradial Oscillations

Described by the real parts of the spherical harmonic functions

$$Y_\ell^m(\theta, \phi),$$

where ℓ is a non-negative integer and m is equal to any of the $2\ell + 1$ integers between $-\ell$ and $+\ell$. There are ℓ nodal circles with $|m|$ of these circles passing through the poles and the remaining $\ell - |m|$ nodal circles being parallel to the equator.



$\ell = 0, m = 0$ (radial)

$\ell = 2, m = \pm 2$

$\ell = 1, m = \pm 1$

$\ell = 2, m = \pm 1$

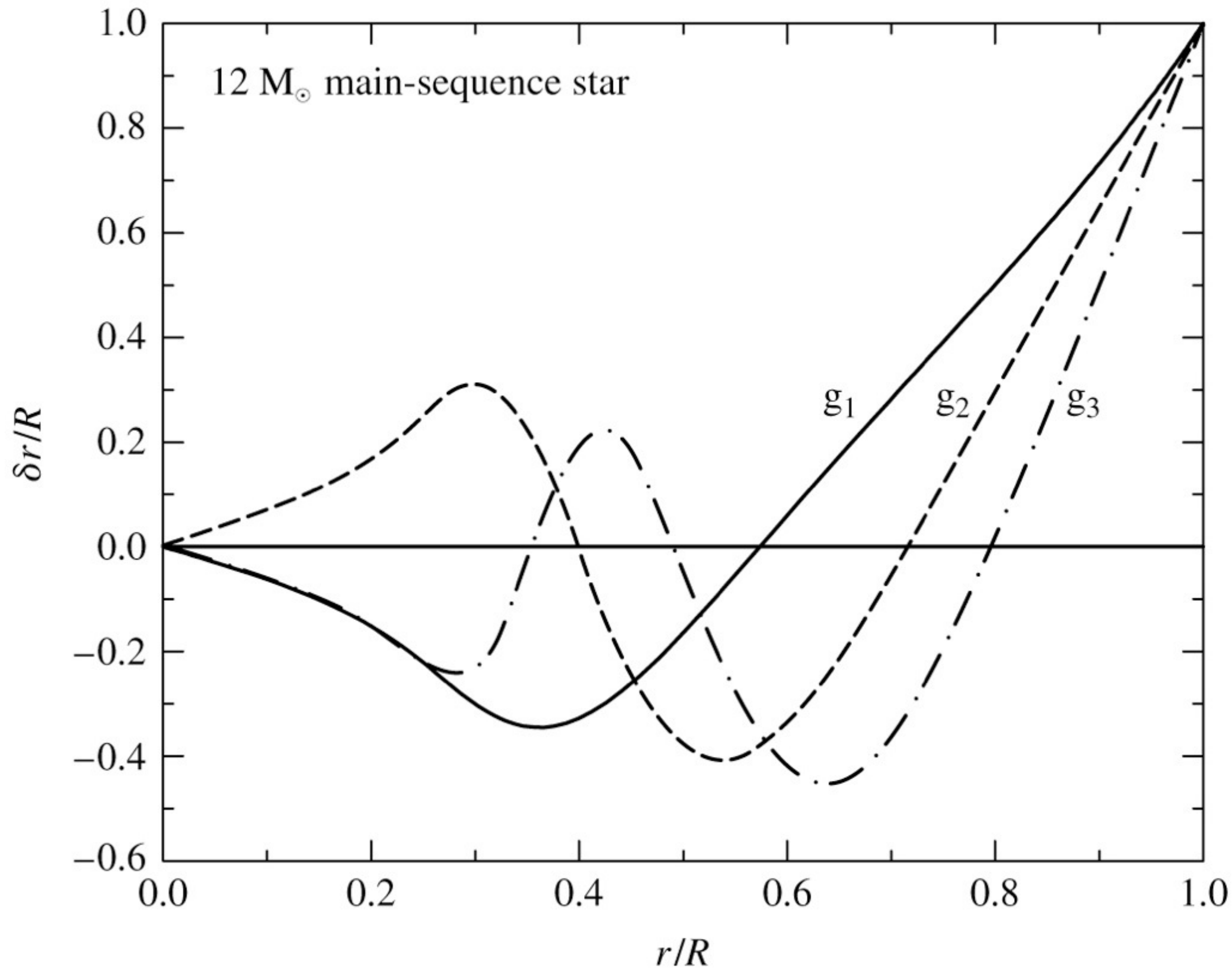
$\ell = 1, m = 0$

$\ell = 2, m = 0$

p, f, and g Modes

- Classified by the restoring forces for sound waves
- May coexist in parts of stellar interior or surface
- Restoring forces
  - Pressure: *p*-mode
  - Gravity (surface gravity waves): *f*-mode
  - Buoyancy (internal gravity waves): *g*-mode

g Modes


12 $M_\odot$ main-sequence star
g_1
g_2
g_3
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
$\delta r/R$
0.0
0.2
0.4
0.6
0.8
1.0
r/R

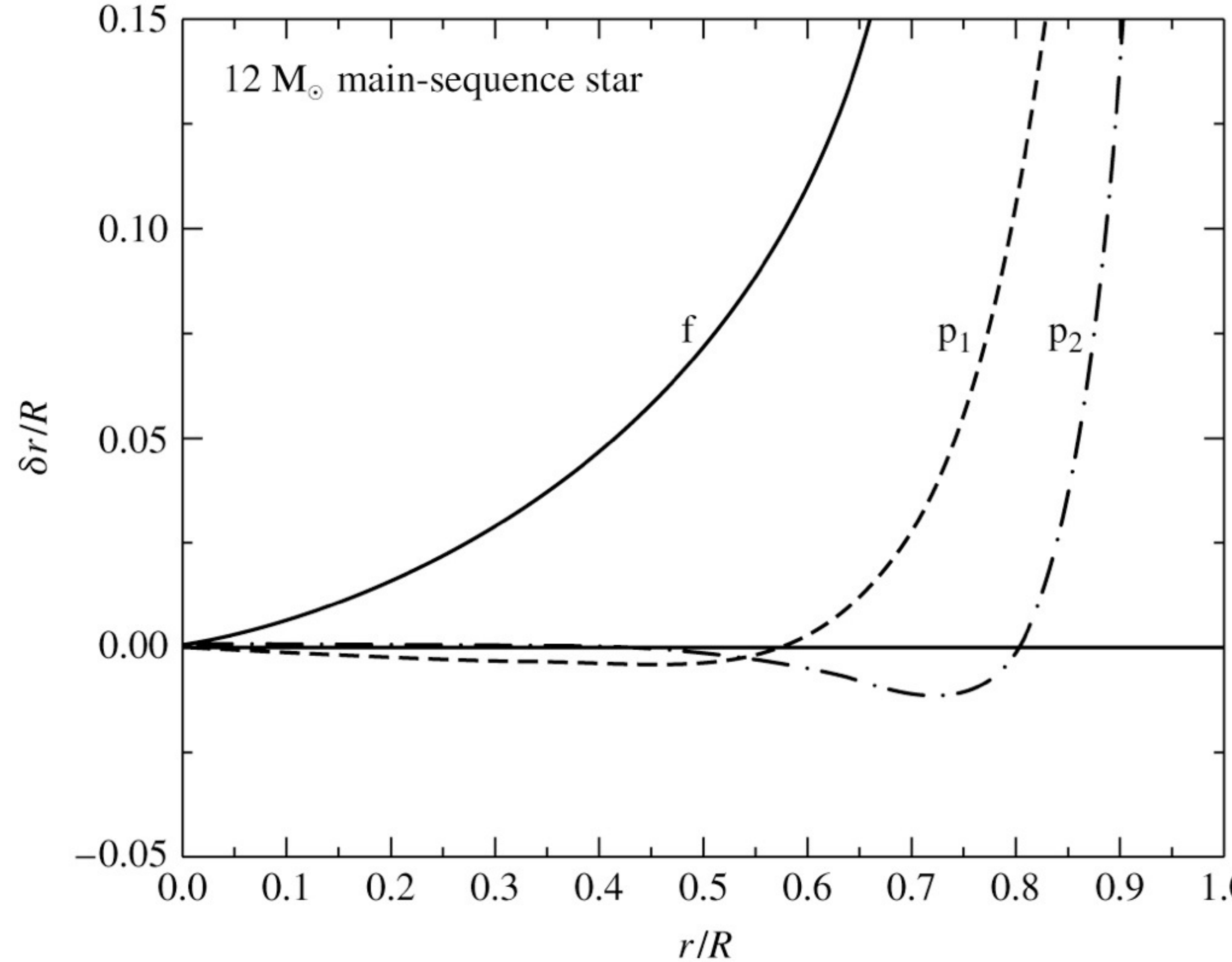
p Modes

Horizontal wavelength is

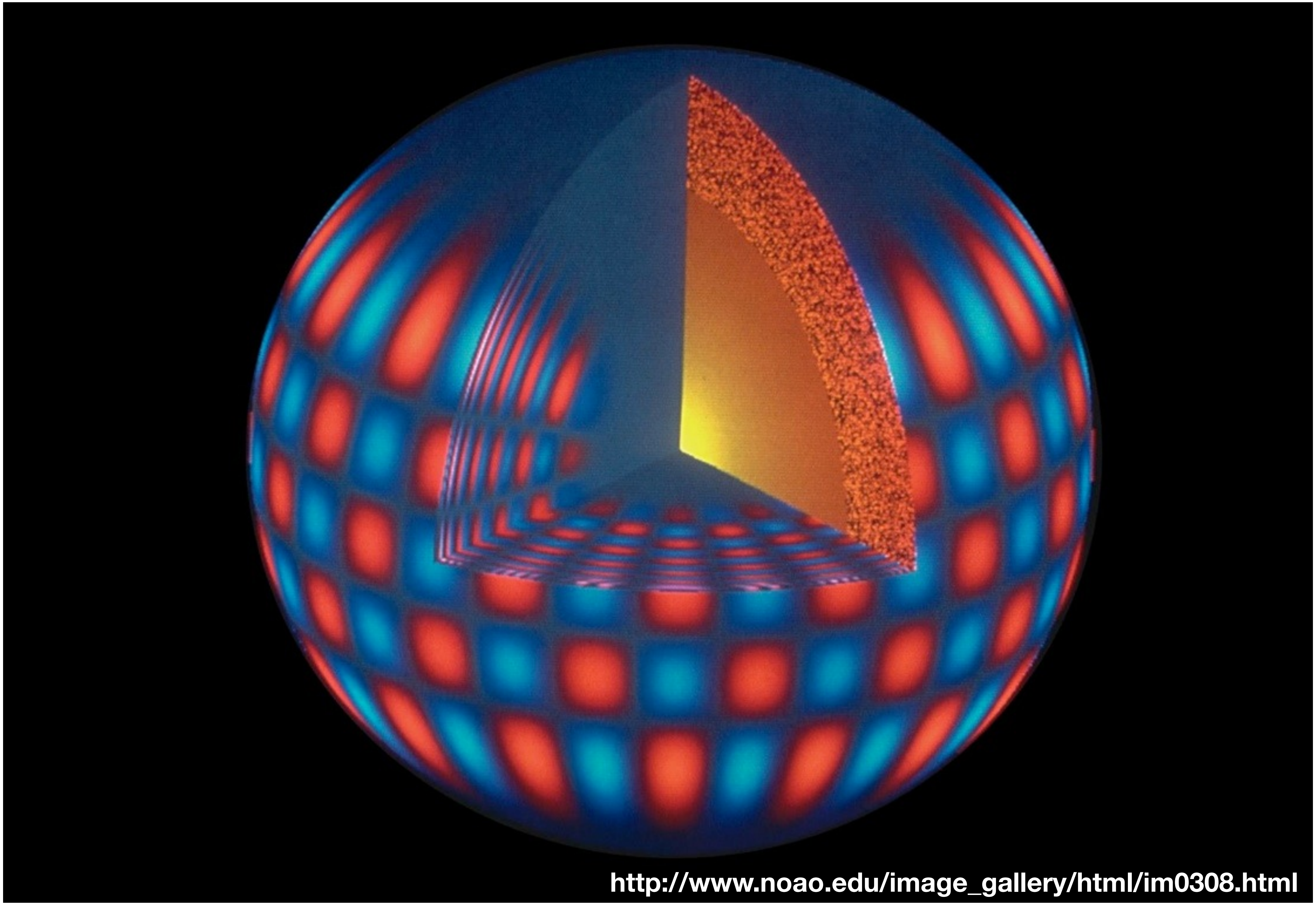
$$\lambda_h = \frac{2\pi r}{\sqrt{\ell(\ell+1)}}.$$

The **acoustic frequency** is

$$S_\ell = \frac{2\pi}{\lambda_h / c_s} = \sqrt{\frac{\gamma P}{\rho}} \frac{\sqrt{\ell(\ell+1)}}{r}$$



Solar Oscillations



http://www.noao.edu/image_gallery/html/im0308.html