Theoretical Mechanics, Spring 2022

# Problem Set VII

**Deadline:** 6PM of Monday, March 14, 2022

1. **Torque on a system (10%).** Even though the total force on a system of particles is zero, the net torque may not vanish. Show that the net torque has the same value in any coordinate system. *Hint:* You may denote the position vectors to the $\alpha$-th particle as $\boldsymbol{r}_\alpha$ and $\boldsymbol{r}'_\alpha$ in the two arbitrary coordinate systems that
$$\boldsymbol{r}_\alpha = \boldsymbol{R} + \boldsymbol{r}'_\alpha.$$

2. **Force law with velocity dependence (10%).** Consider a force law between two particles of the form
$$\boldsymbol{F}(\boldsymbol{r}) = k_1 \left( \boldsymbol{r} - \frac{\dot{\boldsymbol{r}}}{v_0} \right), \tag{1}$$
where $k$ is a constant, and $\boldsymbol{r} \equiv \boldsymbol{r}_1 - \boldsymbol{r}_2$, with $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ being the position vectors of the two particles. Explain why the internal torque does not vanish for a system composed of particles interacting with such force law. Is this a conservative system?

3. **Elastic collisions between two particles (20%).** Consider a particle of mass $m_1$ elastically collides with a particle of mass $m_2$ at rest.

   (a) (10%) What is the maximum fraction of kinetic energy loss for $m_1$?

   (b) (10%) In a pool game, some rules of thumb can be easily understood as elastic collisions between two billiard balls. Explain why two billiard balls often make a right angle (90°) after a collision. Use the result from (a), explain the conditions to make a full stop of the cue ball.

4. **Energies of two-particle collisions (30%).** Consider an elastic collision between two particles of mass $m_1$ and $m_2$, with $m_2$ initially at rest. After the collision, denote the angle of $m_1$ deviated from its initial path as $\psi$ (Fig. 1).

   (a) (15%) Show that the ratio of the kinetic energies $m_1$ before and after the collision, $K_1/K_0$, can be expressed in terms of $m_2/m_1 \equiv x$ and $\cos\psi \equiv y$ as
$$\frac{K_1}{K_0} = \frac{2y^2 + x^2 - 1 + 2y\sqrt{x^2 + y^2 - 1}}{(1+x)^2} \tag{2}$$

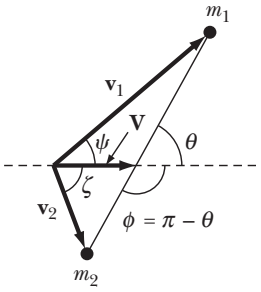


Figure 1:

(b) (15%) Plot $K_1/K_0$ as a function of $\psi$ for $x = 1, 2, 4$, and 12. In the case of $m_1$ being protons or neutrons, what atoms do $m_2$ represent in these energy plots? How about the case of $m_1$ being alpha particles?