Radio Astronomy, Fall 2024

Problem Set II

Deadline: 5PM of Monday, November 4, 2024

extended to 5PM of Wednesday, November 13

1. **Pulse delay times (10%).** Assume that pulsars emit narrow periodic pulses at all frequencies simultaneously. Show that a narrow pulse (width of order $\sim 10^{-6}$ s) will traverse the radio spectrum at a rate of

$$\left(\frac{\dot{\nu}}{\mathrm{MHz\,s^{-1}}}\right) = 1.2 \times 10^{-4} \left(\frac{\mathrm{DM}}{\mathrm{cm^{-3}\,pc}}\right)^{-1} \left(\frac{\nu}{\mathrm{MHz}}\right)^{3}.$$

2. **Temporal smearing of pulses (30%).** The finite bandwidth of a receiver system can cause difficulties to properly differentiate periodic pulses from pulsars.

   (a) (5%) Show that a receiver bandwidth of B will lead to the smearing of a very narrow pulse, which passes through the ISM with dispersion measure DM, to a width

$$\left(\frac{\Delta t}{\mathrm{s}}\right) = 8.3 \times 10^{3} \left(\frac{\mathrm{DM}}{\mathrm{cm^{-3}\,pc}}\right) \left(\frac{\nu}{\mathrm{MHz}}\right)^{-3} \left(\frac{B}{\mathrm{MHz}}\right).$$

   (b) (5%) Show that the ionosphere (electron density $\sim 10^5$ $\mathrm{cm^{-3}}$ and height $\sim$ 20 km) has little influence on the pulse shape at $\nu = 100$ MHz with a bandwidth $B = 10$ MHz.

   (c) (5%) Show that a short pulse suffers from a temporal smearing of

$$\left(\frac{\Delta t/B}{\mathrm{ms\,MHz}}\right) = \left(\frac{\nu}{202\ \mathrm{MHz}}\right)^{-3} \left(\frac{\mathrm{DM}}{\mathrm{cm^{-3}\,pc}}\right)$$

   across the receiver bandwidth.

   (d) (5%) If the pulsar is 5 kpc away and the average electron density is 0.05 $\mathrm{cm^{-3}}$, find the smearing at 300 MHz.

   (e) (10%) Use the result of the previous problems to plot arrival time on the x-axis and frequency on the y-axis for frequencies from 300 to 400 MHz. First, sketch the shape of a pulse after it traverses the ISM with DM = 10 $\mathrm{cm^{-3}}$ pc. Repeat for pulses spaced by ± 50 ms. From this sketch, consider whether the detection in a band covering 50 MHz, centered at 350 MHz, is confused by the simultaneous arrival of two separate pulses.

3. **Rotation Measure in the terrestrial ionosphere (15%)**. Determine the *upper limit* of the angle through which a linearly polarized electromagnetic wave is rotated when it traverses the ionosphere. Assume an ionospheric depth of 20 km, an average electron density of 10^5 cm^{-3} and a magnetic field strength (parallel to the direction of wave propagation) of 1 Gauss.

   (a) (5%) Find the rotation measure (RM).

   (b) (5%) Carry out the calculation for the Faraday rotation, $\Delta\psi$, for frequencies of 100 MHz, 1 GHz, and 10 GHz, if the rotation is $\Delta\psi/\mathrm{rad} = (\lambda/\mathrm{m})^2\,\mathrm{RM}$.

   (c) (5%) What is the effect if the magnetic field direction is perpendicular to the direction of propagation? What is the effect on circularly polarized electromagnetic waves?

4. **Rotation Measure in the solar system (15%)**. Determine the *upper limit* of the angle through which a linearly polarized electromagnetic wave is rotated when it traverses the solar system. Let the average charged particle density in the solar system is 5 cm^{-3}, the magnetic field 5 μG and the average path 10 AU.

   (a) (5%) Find the RM.

   (b) (5%) What is the maximum amount of Faraday rotation for frequencies of 100 MHz and 1 GHz?

   (c) (5%) Must radio astronomical results be corrected for this effect?