Radio Astronomy, Fall 2024

Problem Set IV

Deadline: 5PM of Monday, December 2, 2024

extended to 5PM of Wednesday, December 11

1. **Thermal emission from warm dust (35%).** The Orion hot core is a molecular source with an average gas temperature of $T_{\rm gas} = 160$ K, angular size of $10''$, and a distance of $d = 0.5$ kpc. The average gas density is $n_{\rm H_2} = 10^7$ cm^{-3}.

   (a) (5%) Assume the region to be of uniform density and spherical geometry. Calculate the column density, $N_{\rm H_2}$.

   (b) (10%) Obtain the flux density at 1.3 mm, $F_{1.3\ \rm mm}$, by assuming thermal equilibrium between gas and dust ($T_{\rm dust} = T_{\rm gas} = 160$ K), an opacity spectral index of $\beta = 2$, and a gas-to-dust mass ratio of 100. Use the dust opacity law

   $$\kappa_\nu^{\rm dust} = 10\ {\rm cm^2\,g^{-1}} \left(\frac{\lambda}{250\ \mu{\rm m}}\right)^{-\beta}.$$

   (c) (5%) Use the Rayleigh-Jeans approximation to obtain the dust continuum brightness temperature T_b for the given $F_{1.3\ \rm mm}$. Show that $T_b < T_{\rm dust}$ and determine $\tau_{\rm dust}$ by assuming optically thin emission, $T_b = \tau_{\rm dust} T_{\rm dust}$.

   (d) (5%) At what wavelength is $\tau_{\rm dust} = 1$ when $T_b \propto \nu^4$.

2. **Bremsstrahlung Emission (25%).** The SEDs of the Orion A H II region is given in Fig. 1. Assume $a(\nu, T) = 1$ and answer the following questions.

   (a) (5%) Determine the turnover frequency, $\nu_{\rm TO}$, of the Orion A H II region. Given this turnover frequency, $\nu_{\rm TO}$, determine the emission measure, EM, for an ideal H II region with uniform density and constant electron temperature of $T_e = 8.3 \times 10^3$ K.

   (b) (10%) A more accurate method to obtain the EM is to derive the optical depth through the brightness temperature $T_b = \tau_{\rm ff}\, T_e$. In the case of Orion A, one observes $T_b = 26$ K at $\nu = 23$ GHz. Determine the optical depth, $\tau_{\rm ff}$, turnover frequency, $\nu_{\rm TO}$, and EM for Orion A.

   (c) (10%) For frequencies above 2 GHz, the emission of Orion A appears to be optically thin and $\tau_{\rm ff} \propto \nu^{-2.1}$. Compute the expected optical depth, $\tau_{\rm ff}$, and

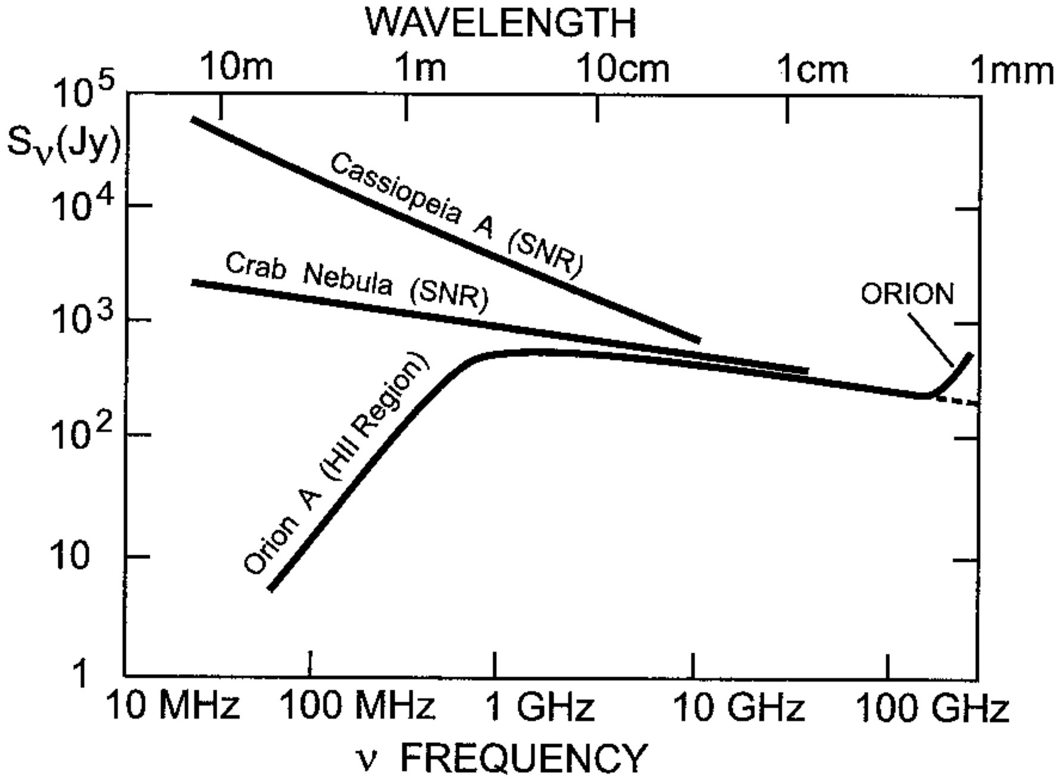

Figure 1: Spectra Energy Distributions (SEDs) of the H II region Orion A, and the Crab nebula. The flux density of Orion above 100 GHz is caused by dust emission from the Orion KL region which is 45″ northwest to the center of Orion A and behind.

brightness temperature, T_b, at $\nu = 5$, 10, 23, 90, 150, and 230 GHz.