Stellar Astrophysics, Fall 2024

Problem Set III

Deadline: 5PM of Thursday, November 7, 2024

1. **The Saha equation (15%).** A stellar atmosphere consists almost entirely of hydrogen. Assume that 50% of the hydrogen molecules are dissociated into atoms and that the pressure is 10^3 dyn cm^{-2}. Estimate the temperature given that the binding energy of the hydrogen molecule is 4.48 eV and the ratio of degeneracy to be $g(\mathrm{H}_2)/g(\mathrm{H}) \sim 1$.

2. **Equilibrium of pair production (15%).** In the hot central regions of an evolved star, the gas is considered to form with dilute, classical positrons and dense, degenerate electrons. Consider the electron-position production

$$\gamma + \gamma \rightleftharpoons e^+ + e^-$$

in such a degenerate electron gas with Fermi energy ϵ_F.

   (a) (5%) Find the chemical potential for a classical gas of non-relativistic positrons as well as that for a *degenerate* gas of non-relativistic electrons.

   (b) (5%) Derive an expression for the equilibrium concentration of positrons using $\mu_{e^+} + \mu_{e^-} = 0$.

   (c) (5%) Make a numerical estimate for n_{e^+} in stellar matter at $T = 10^9$ K and $\rho = 10^7$ g cm^{-3}.

3. **Photodisintegration of ^{4}He (15%).** When the core of a massive star exceeds the Chandrasekhar limit, it collapses. During this collapse, energy is absorbed by the photodisintegration of ^{4}He via the reaction

$$\gamma + {}^4\mathrm{He} \rightarrow 2p + 2n.$$

The energy required for this reaction is $Q = 28.30$ MeV. Assume that this reaction is in equilibrium with its inverse. Estimate the temperature at which 50% of the ^{4}He is dissociated into nucleons when the density is 10^9 g cm^{-3}.