Stellar Astrophysics, Fall 2024

Problem Set IV

Deadline: 2PM of Monday, November 18, 2024

1. **Kramers' law (10%).** Show that, if the frequency and temperature dependence of the mean free path for a photon is given by

$$\bar{\ell}_\nu \propto \nu^3 T^{1/2},$$

then the frequency averaged opacity, Rosseland mean opacity, satisfies Kramers' law

$$\kappa \propto \rho T^{-3.5}.$$

2. **Eddington luminosity (15%).** In extremely massive main-sequence stars with mass on the order of $\gtrsim 100 M_\odot$, the radiation pressure can be strong enough to blow away the stellar atmosphere.

   (a) (5%) Show that heat transfer by radiative diffusion implies a non-zero gradient for the radiation pressure, which is proportional to the radiant heat flow.

   (b) (5%) Recall that the magnitude of the force per unit volume in a fluid due to the pressure is equal to the pressure gradient. Find the radiant heat flux density which can, by itself, support the atmosphere of a star with surface gravity g. Hence show that a star of mass M has maximum luminosity given by

$$L_{\max} = \frac{4\pi c GM}{\kappa},$$

   where κ is the opacity near the surface. This maximum luminosity is usually referred to as the Eddington luminosity, which sets the upper mass limit of main sequences.

   (c) (5%) Obtain a numerical estimate for this luminosity to mass ratio in units of $L_\odot/M_\odot$ by assuming that the surface is hot enough for the opacity to be dominated by electron scattering.

3. **Criterion for the onset of convection (10%).** Recall that the adiabatic index, γ, is the ratio of the heat capacities at constant pressure and at constant volume, $\gamma \equiv C_P/C_V$. Show that, for an ideal gas of classical particles, the critical temperature for the onset of convection,

$$\frac{\mathrm{d}T}{\mathrm{d}r} < \frac{\gamma - 1}{\gamma}\frac{T}{P}\frac{\mathrm{d}P}{\mathrm{d}r},$$

can be written as

$$\left.\frac{\mathrm{d}T}{\mathrm{d}r}\right|_{\mathrm{conv}} = -\frac{g}{C_P},$$

where C_P is the heat capacity at constant pressure and g is the acceleration due to local gravity. Note that when the heat capacity is high because of the absorption of heat by the excitation and/or the dissociation of the constituent particles, the temperature gradient needed for convection is less steep.