Stellar Astrophysics, Fall 2024

Problem Set V

Deadline: 5PM of Thursday, November 21, 2024

1. **Outer envelopes of white dwarfs (15%).** In class, we showed that the pressure and temperature in the outer envelope of a white dwarf are related by

$$\frac{P^2}{2} = \left[\frac{16\pi}{3}\frac{ac\,Gk}{\kappa_0\,\overline{m}}\frac{M}{L}\right]\frac{T^{8.5}}{8.5}.$$

   (a) (10%) Show that the radiative temperature gradient in the outer envelope is given by

$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{GM\overline{m}}{4.25\,r^2\,k}.$$

   (b) (5%) Consider a white dwarf with mass $M = 0.4M_\odot$ and radius $R = 0.01R_\odot$ with an internal temperature of 10^7 K. Estimate the thickness of its outer envelope.

2. **Cooling time of white dwarfs (10%).** Show that the time for a carbon white dwarf of mass M to cool from a high internal temperature to a much lower internal temperature, T_I, is approximately

$$t = \frac{3}{5}\frac{kT_I}{L}\frac{M}{12m_\mathrm{H}},$$

   where L is the luminosity corresponding to T_I.